DUFF & PHELPS

Oil & Gas Intelligence Report

September 2017

CONTENTS

A DISCUSSION OF PRICE FORECASTING METHODOLOGIES

Throughout this report, Duff & Phelps will analyse the nature of crude oil prices, their historical evolution and the factors that condition their changes in order to evaluate certain tools for their prediction.

As observed during the last decades, oil prices, mainly because of the influence of exogenous factors, have shown significant oscillations that have created a frame of uncertainty that may not be easy to manage.

Under these circumstances, the first important conclusion drawn from this report is that it will be more feasible and reliable to estimate future oil price volatility than to directly predict prices. The underlying reason is the type of time series that defines crude oil prices. In this sense, in the present study, Duff & Phelps will develop forecasting models for absolute price forecasts and future estimates of their volatility.

Prices are observations of a variable in time – that is, time series. Time series are moved according to three components: trend and seasonality, as well as a factor of irregularity or randomness of these two components.

Depending on the behaviour of each component, time series will be classified as stationary or non-stationary, with non-stationary being those with a changing trend (mean) and a seasonality (variance). In this regard, another concept will be added: heteroscedasticity, which defines models with changing volatility. The heteroscedastic models best meet the requirements of the study of the current movements of the crude price.

In addition, and as a starting point of the present report, Duff & Phelps will verify how the trend pattern has changed throughout the life of this commodity, going from a mean reverting pattern to a completely random one, statistically defined by a Browinian Motion.

Regulatory Frame of Reference

Every year-end, the Securities and Exchange Commission (SEC), through documents 17 CFR parts 210 and 211 “Modernization of Oil and Gas Reporting; Final Rule” of January 14, 2009, requires Oil & Gas companies to calculate the value of their proven reserves (deterministic calculation), for subsequent verification, as a means to control and homogenise the available information to investors and shareholders. For this calculation, the SEC determines a fixed price, commonly known as price deck.

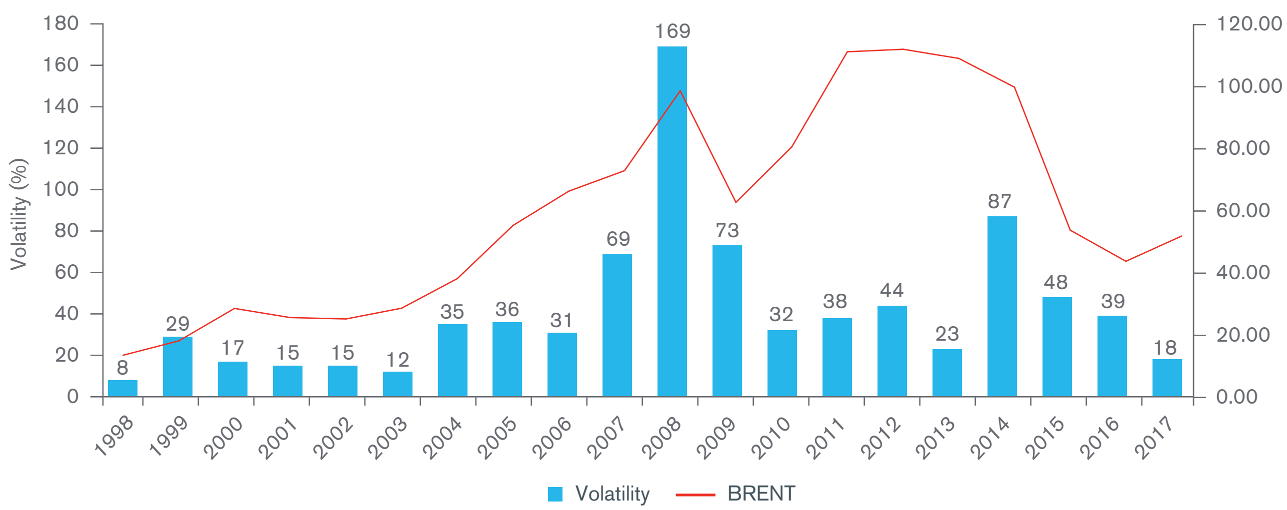
The SEC methodology to determine the price deck has changed in recent years. Between 2006 and 2008, the price benchmark was that of the same day of the prior year, while from 2009 onwards, the price has been determined by obtaining the average price of the first day of the month throughout the whole year in an attempt to avoid or minimise the volatility of the full year.

The Difficulty of Estimating Future Oil Prices

The prevailing idea along this study is the impossibility of performing solvent price forecasts because of the diversity of influence factors acting at the same time, managed by different parties with diverse interests and, in many circumstances, with little or nothing to do with the actual dynamics of the Oil and Gas business. Geopolitical control or financial speculation matters, mixed with the typical adjustment mechanisms of supply and demand, might make the oil price estimation process an unsolvable puzzle in which the pieces change constantly in size and theme.

As it is possible to observe in Figure 1, developments in volatility of oil prices are extraordinarily variable and unpredictable. Throughout the 2000–2015 period, volatility reached its peaks in 2008 and 2014, although since 2000, figures expressing this parameter diverged significantly from historic data. This reinforces the theory of change in the trend in oil prices from mean reverting to Brownian Motion.

Figure 1: Brent price evolution and volatility (average price in USD)



Average Price of the year (USD)	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	June 2017
BRENT	13.3	17.9	28.4	25.4	25.0	28.4	37.9	55.0	66.0	72.6	98.3	62.4	80.2	110.8	111.7	108.7	99.5	53.5	43.5	51.6
Annual Minimum								44.4	59.8	54.6	43.0	43.8	74.7	96.9	95.9	103.2	63.2	38.0	30.7	46.3
Annual Maximum								63.8	74.2	92.2	134.5	77.5	92.2	123.0	124.5	116.0	111.9	65.6	53.2	54.8
Range								19.3	14.4	37.5	91.5	33.7	17.4	26.1	28.6	12.8	48.7	27.6	22.5	8.5
Annual Standard Deviation	4.9	17.5	10.0	9.1	9.0	7.2	20.5	21.4	18.7	40.9	100.9	43.2	18.9	22.8	26.4	13.6	51.8	28.4	23.5	10.9
Volatility Change	8%	29%	17%	15%	15%	12%	35%	36%	31%	69%	169%	73%	32%	38%	44%	23%	87%	48%	39%	18%

So what happens in these periods when volatilities of crude oil, and commodities in general, are so high?

During the 1997 Asian crisis (dragon effect), several economies had low levels of international reserves, which they had to compel with harsher terms imposed by the International Monetary Fund, thus affecting market confidence significantly. From that moment, Asian governments intervened to keep the value of their currencies low, increasing the appetite for commodities and introducing the idea of economic growth achievement by relying on exports, while at the same time building stockpiles of international reserves in order to cover future crises.

The first decade of the 2000s witnessed an increase in commodity prices. On January 2008, soaring oil prices breached the barrier of US$100 per barrel for the first time in history, rising to US$147 per barrel in July because of speculative phenomena which ended in a sharp drop during the month of August.

The U.S. current account deficit grew substantially. Nevertheless, until 2007, general consensus amongst free-market economists and policymakers such as Alan Greenspan, chair of the Board of Governors of the Federal Reserve at the time, and Paul O'Neill, secretary of the Treasury of the United States, was that the deficit was not an issue to be concerned about.[1]

Until 2014, experts increased warnings over a possible currency war in the future. This time, instead of being aimed at driving competitiveness, some states, particularly Japan and the Eurozone, could be targeted to devalue their currencies as a means to deter deflation.

Summarising, it is possible to intuit a great deal of the challenge when estimating future oil prices lies in determining a solvent and sustainable volatility parameter for the long run.

Historical Price Trend Analysis

The investigation of the accuracy of oil price forecasts is of vital importance for the world economy and one of the biggest challenges for economists and mathematicians, considering direct influence in different variables and macroeconomic events such as inflation, GDP, investments or recessions.[2]

Commodity prices are extremely volatile, as explained by Deaton and Laroque in 1999.[3] Both authors established that, in general, price series do not show a clear trend, even if analysed over an extensive period of time; they display sudden leaps, which disappear quickly.

In oil price analysis, two main streams of thought exist: on one side, there are those who believe oil has always behaved as a mean-reverting process, and on the other side, there is a group which thinks it follows a Geometric Brownian Motion[4] – namely, a Wiener process.[5]

From reading several published studies, Duff & Phelps can conclude that there is not a single opinion, at least regarding the periods in which crude has followed one trend or another.

In this regard, the analysis of historical oil price trends begins with the reading of the study published by Delson Chikobvu and Knowledge Chinhamu,[6] which concludes that oil prices have not historically evolved under the same statistical pattern. According to the aforementioned authors, in the period between 1980 and 1994, oil prices followed a mean-reverting trend, while since 1994 up to now, the behaviour has been that of a Brownian Motion. Therefore, Duff & Phelps can state that if crude oil prices continue to behave randomly in the future, predicting their movements will be inviable.

Another consulted study was German's work (2007). German used a simple regression model in order to determine if crude oil prices had complied with a mean-reverting model during the 1994–2004 period:

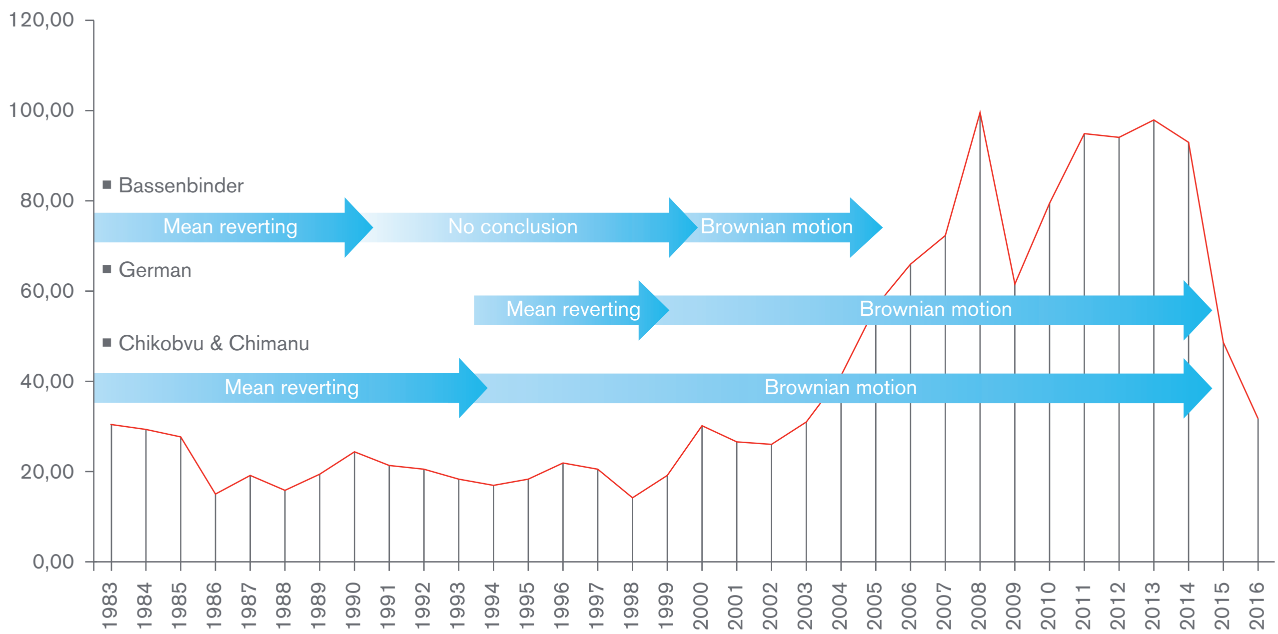
$$P_t = \varphi P_{t-1} + \varepsilon_t \qquad \text{(Equation 1)}$$

with P_t being the oil price logarithm. In this way, if φ differs significantly from 1, then the mean-reverting hypothesis would be rejected. The result was 0.651, thus Duff & Phelps can assert that the variable does not follow a mean-reverting process, at least for the full length of the analysed time span, and that it did for a shorter period of time, particularly in the years between 1994 and 2000.

Bassembinder (1995) analysed crude price trends compared to traded futures in the market, determing that an inverse relationship between both prices constitutes evidence of mean reverting, as investors discount price drops when spot prices are higher. His conclusions led to the existence of a period of mean reverting between 1982 and 1991. Despite this, the same calculations were inconclusive for the period between 2000 and 2005.

Other consulted sources include Pindyck (1999) and Bernard (2008). In both cases, opinions have been diverse regarding the fulfilment of a stochastic process in certain periods.

Figure 2: Experts' trends analyses



Absolute Price Estimation Models

GEOMETRIC BROWNIAN MOTION WITH TREND OR "WIENER PROCESS"

Through the Wiener process and the mean-reverting model, Duff & Phelps will analyse price forecasts from an absolute perspective.

A geometric Brownian model is a stochastic process composed by a deterministic part (first addend) and a random part (second addend). Where Z is a random variable with a mean of zero and a standard deviation of one,

$$dx = dt + \sigma dZ \qquad \text{(Equation 2)}$$

To properly comprehend this, Duff & Phelps will reflect on several related concepts:

a. Martingale Process[7]

In probability theory, a martingale stochastic process is any process characterised by having no stochastic drift or tendency. The concept was immediately applied to the analysis of stock prices. One of the most relevant results obtained from financial mathematics is precisely that a perfect market without arbitrage is a martingale. The most distinctive example of martingale-type stochastic processes is Brownian Motion, and this aspect is one of the foundations of Lévy's Theorem, which states, "A continuous martingale process is a Brownian Motion if and only if its quadratic variations in any time interval [0, T] are equivalent to T".

b. Itô Process[8]

An Itô process is an "X" variable that changes over time. From Equation 2 we can infer:

$$dX(t) = (t)dt + \sigma(t)dZ(t) \qquad \text{(Equation 3)}$$

In the formula, μ and σ may also follow a random process, but if both are constant, the Itô process is called Brownian Motion. If $\mu \neq 0$, it will not be a martingale, but a Wiener process.

c. Geometric Brownian Motion

A Brownian Motion is a random process that evolves continuously and has the properties of a normal distribution – that is, a mean of zero and a standard deviation of one (therefore, an Itô process and a martingale).

A Brownian Motion is composed of a deterministic part (the drift of a martingale process) and a stochastic part:

$$dx = a\,dt + b\,dZ \qquad \text{(Equation 4)}$$

Recalling the explanation of an Itô process on the values taken by μ and σ, to be a martingale, this being a standard Brownian Motion, the mean must be zero and the standard deviation one. If $\mu \neq 0$, it would not be a martingale but a Brownian Motion with trend or Wiener process.

d. Wiener Process

This is a type of stochastic process that is continuous in time, characterised by having two important attributes:

(i) It is a Markov process:[9] Thus the probability distribution of all future values depends only on their current value and is not affected by historical values or any other current information. Therefore, the current value of the process is the only necessary information to make the best estimate of its future value. In this way, for example, it is determined that current prices collect all future information from the markets.

This attribute, as has been noted, is fully applicable to the current situation in the oil market.

(ii) Increments are independent: If random variable "Z" follows a Wiener process, variations (Δz) are independent and follow a normal distribution, with a variance that increases linearly with the size of the time interval. Therefore, $\Delta z=\varepsilon\sqrt{\Delta t}$ (where ε is a random variable of type ϕ [0,1]).

Thus, and in keeping with the central limit theorem, the Wiener process is obtained as the limit of the sum of identically distributed independent random variables, therefore being a normal distribution.

A Wiener process has a trend per unit time of μ and a standard deviation σ or, otherwise said and according to Equation 4, follows a normal distribution with mean equal to "μdt" and a standard deviation of "σdZ".

A Wiener process is a Brownian Motion with trend used to model the returns of assets. Consider that Equation 5 is equivalent to Equation 4:

$$dLn\,S(t) = (-\frac{1}{2}\sigma^2)dt + \sigma dZ(t) \qquad \text{(Equation 5)}$$

or expressed in discrete terms

$$S_t = S_o e^{[-\frac{\sigma^2}{2}T+\sigma Z\sqrt{T}]} \qquad \text{(Equation 6)}$$

where the trend of movement is $(\mu-\frac{1}{2}\sigma^2)$ dt, which at that practical level should be replaced by the minimum return required by investors.

Duff & Phelps will select the annual Brent prices from January 1988 to December 2015 as the sample to analyse, calculating the average and standard deviation, which correspond to US$45.71 per barrel and 34% annually, respectively. Next, a linear regression will be developed, which will result in a = 6.67 and b = 0.89.

As it will be explained later, investors in the oil industry demand an average rate of return of 10%.[10] The significant conclusion that can be derived from this calculation is that, according to an average price of approximately US$46 per barrel, investors will get a profit premium of almost 3% (10% – 6.67%). In other words, under a price scenario of US$47 per barrel, and an effective use of resources, a standard oil extraction project must be profitable.

Duff & Phelps will apply Equation 6 considering the following parameters:

- Brent price returns since 1998
- $\mu = 8.4\%$
- $\sigma = 28.2\%$
- Initial starting price in the Markov process: US$40 per barrel[11]

Under these premises, price estimates under a Wiener model would be as follows (see Figure 3):

Figure 3: Weiner estimation

	2017	2018	2019	2020	2021
N(0,1) = Z	(1.304)	(1.749)	1.965	0.107	0.314
Wiener	0.715	0.631	2.254	1.096	1.257
Estimation	**56.8**	**49.8**	**47.5**	**68.9**	**51.7**

MEAN-REVERTING MODEL

Commodity prices, especially oil, have amongst their “supposed”[12] properties a mean-reverting and a stochastic trend.[13] This mean reverting is explained as prices are expected to converge towards the value of the long-term marginal cost of extraction per barrel of crude oil.

In general, the behaviour of crude oil price variations will be represented by a Geometric Brownian process according to the following equation:

$$X_{t+1} = X_t + k(\mu - X_t) + \sigma\varepsilon_t, \text{ For } t = 0\ldots n \qquad \text{(Equation 7)}$$

- X_t: Current price and the base of the model
- μ: Long-term average, where prices will converge
- k: Mean-reverting speed. The obtained value must be between 0 and 1, where 1 indicates an instantaneous adjustment, while if the parameter tends to zero, the setting speed reverting to mean will be slower
- σ: Volatility of the price of the asset
- εt: Represents a process which is also random, tending towards a normal distribution N (0,1). White noise

Subsequently, Duff & Phelps will calculate the parameters μ and k with a linear regression, according to least squares equations, where the following equivalence will be established:

Equation 7 may be expressed as

$$X_{t+1} = b\,k + (1 - b)\,X_t + \varepsilon_t \qquad \text{(Equation 8)}$$

or stated otherwise as

$$X_{t+1} = \alpha + \beta\,X_t + \varepsilon_t \qquad \text{(Equation 9)}$$

Therefore, and from the previous equivalence, it can be observed that using the same time interval (1998–2015), once the regression has been calculated, the following results will be obtained:

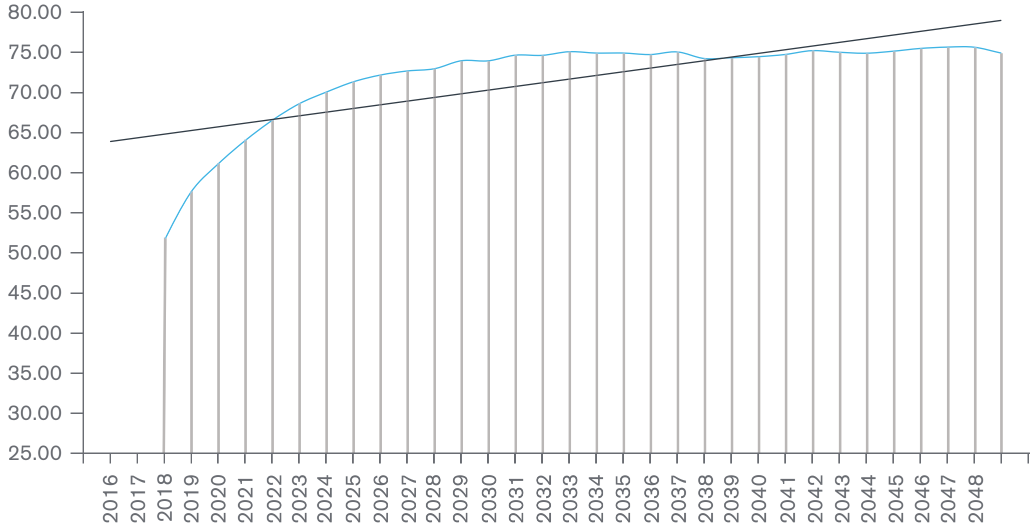
- α = 16.84
- β = 0.78
- μ = Average prices (1998–2015) = US$61.02 per barrel
- k = 22.5% (k = 1 – b)

From these data, crude oil prices for the coming years will be as follows (see Figures 4 and 5):

Figure 4: Mean-reverting estimation

	2017	2018	2019	2020	2021
Mean Reverting Estimation	**51.7**	**56.8**	**61.2**	**64.5**	**66.7**

Figure 5: Annual Brent prices with mean reversion



As Duff & Phelps has already pointed out in this study, the mean reverting in oil may not be fully clear, as Hélyette Geman states in her study.[15] The study reveals the existence of two periods of time: (i) from 1994 to 1999, a clear mean-reverting pattern can be observed, and (ii) from the year 2000 onwards, it changes to a Brownian Motion. Therefore, when applying mean reverting to the simulation of future oil prices, although interesting, it must be carried out with caution and consideration of complementary alternatives.

POLYNOMIAL REGRESSION MODEL

Simple regression models are based on the simple and "automatic" idea of value estimation of the variable in a point in time depending on its value immediately before that moment, through the interpretation of the series according to a certain mathematic function, generally additive or multiplicative.

In general, when dealing with oil price estimations, whether of financial assets or commodities, it is logical to assume that variations in prices (yields) follow a lognormal distribution, as prices cannot be below zero. In this regard, changes in the logarithms of prices will distribute according to a normal distribution.[16]

$$u_t = \mathrm{Log}\,(R_t / R_{t-1}) \qquad \text{(Equation 10)}$$

The selection of logarithms, on the other hand, reduces the affect of heteroscedasticity, which occurs when long series with high frequencies are chosen:

$$u_t = \beta_0 + \beta_1 U_{t-1} + \varepsilon_t \qquad \text{(Equation 11)}$$

where $\varepsilon_t \sim N(0, \sigma^2)$.

From this point, Duff & Phelps will state the equation depending on the error term (ε^t):

a) On one side, it is possible to define εt as an independent term that is identically distributed over time, with zero mean and constant variance σ^2. In this case, Duff & Phelps would be talking about a *homoscedastic model*.

b) If, on the other side, ε_t is an independent term but is not identically distributed over time, since the variance is σ_t^2, Duff & Phelps would be speaking about a heteroscedastic model.[17]

In the calculation of simple regression models, a polynomial equation of the third degree will be used, defining the historical series (2005–2015) in the first place. The first obstacle will be to obtain reasonable conclusions, in respect of the length and fluctuation of the series. In this case, the weighted moving average (WMA) method will be useful.

Figure 6: Brent with polynomial regression

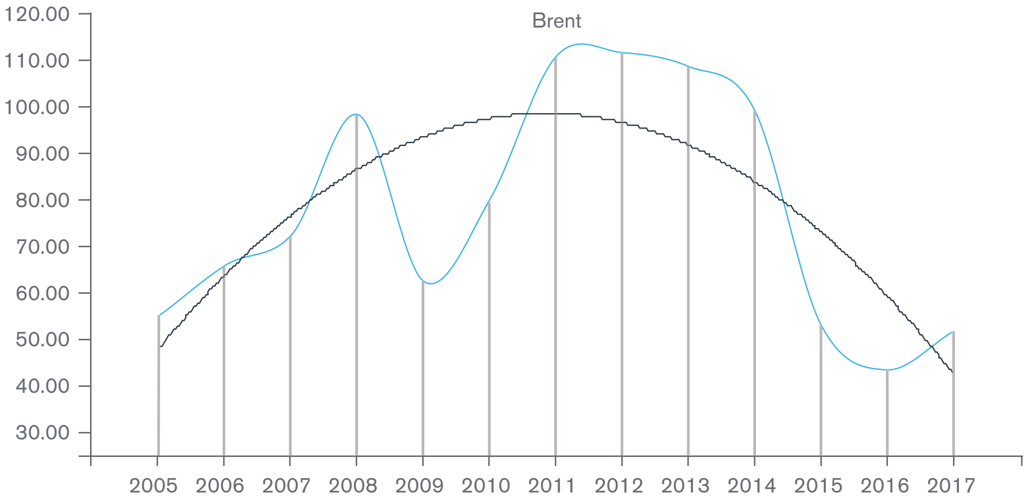


Figure 7: Polynomial estimation

	2017	2018	2019	2020	2021
Polynomial Estimation	**55.7**	**64.1**	**72.9**	**81.4**	**89.0**

Volatility Estimation Models

It is possible to distinguish between different approaches in the study of time series depending on the type of volatility analysis, as it can be historical or implied. In this article, Duff & Phelps will develop both approaches, analysing in the first both homoscedastic and heteroscedastic models. Within historical volatilities, simple regression and autoregressive homoscedastic (ARIMA)[18] models will be applied, and included in the second approach, Duff & Phelps will use exponential weighted moving average (EWMA) and autoregressive heteroscedastic (ARCH)[19] models. For the analysis of implied market volatility, Duff & Phelps will present futures market quotations.

HOMOSCEDASTIC MODELS

ARIMA Models
When a statistical development is made from a stochastic stationary process of variance at constant rates, series can be described with ARIMA models.

In 1970, Box and Jenkins developed a methodology in order to identify, estimate and diagnose dynamic models of time series in which the variable time plays a fundamental role.

Duff & Phelps defines a model as autoregressive if the endogenous variable of a period “t” is explained by its own observations, corresponding to previous periods, adding, as in the structural models, an error factor. In the case of stationary processes with normal distribution, statistical theory of stochastic processes says that under certain preconditions, all Y_t can be expressed as a linear combination of its historical values (systemic part) plus an error term.

Autoregressive models are often abbreviated with “AR” after which the order of the model is indicated: AR (1), AR (2), etc. The order of the model expresses the number of delayed observations, of the analysed time series, that intervene in the equation.

The error term of this type of model is generally denominated as “white noise” when it fulfils the three aforementioned traditional basic hypotheses: (i) null mean, (ii) constant variance and (iii) null covariance between errors corresponding to different observations.

Assuming that this type of model does not work properly with non-stationary series, especially with high volatilities, Duff & Phelps will estimate a five-year price forecast model, obtaining the following results (with very wide divergences; Figures 8 and 9):

Figure 8: ARIMA

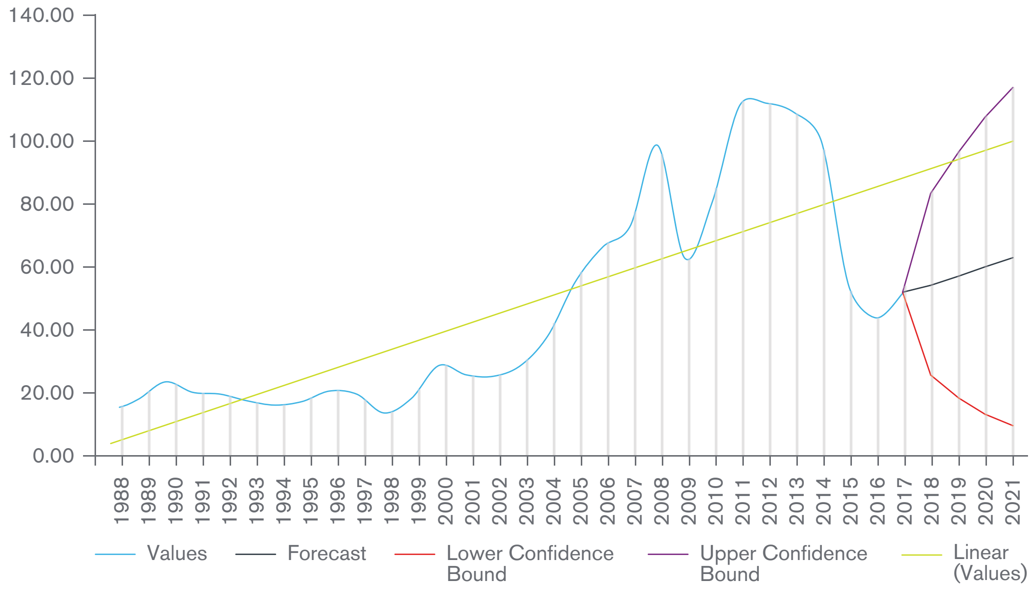


Figure 9: ARIMA estimation

	2017	2018	2019	2020	2021
ARIMA	**51.7**	**54.2**	**57.0**	**59.9**	**62.8**

HETEROSCEDASTIC MODELS

Volatility is an inherent characteristic of financial time series. In general, it is not constant, and, in consequence, traditional time series models with homoscedastic variance are not adequate for the development of forecast models.

Moving Weighted Average

Amongst the exponential smoothing models, the most interesting are those of moving average, both weighted (WMA)[20] and exponential weighted (EWMA)[21] models, which owe their names to the greater relevance given to recent data compared to older data.

The smoothing techniques are focused on the EWMA and follow two steps:

1. Calculate the series of natural logs of periodic returns.

2. Apply a weighting scheme.

That leads us to the second step, and this is where the two approaches (WMA and EWMA) differ.

The simple variance is the average of the squared returns:

$$\text{Variance} = \sigma_n^2 = \frac{1}{m}\Sigma_1^m U_{n-1}^2 \qquad \text{(Equation 12)}$$

Notice that this sums each of the periodic returns and then divides that total by the number of days or observations (m). Therefore, it is just an average – that is to say, each squared return has an equal weight, so if alpha (a) is a weighting factor (specifically, $a = \frac{1}{m}$), then a simple variance looks like Equation 12.

Exponential Moving Weighted Average

The EWMA introduces lambda (l), which will be the smoothing parameter. L must be less than one. Under that condition, instead of equal weights, each squared return is weighted by a multiplier as follows:

$$\text{Weight: } (1-l)1^{0},\ (1-l)l^{1},\ (1-l)l^{2}\ldots(1-l)l^{n} \qquad \text{(Equation 13)}$$

A WMA model assigns a weighting factor to each value in the data series according to its age. The most recent data gets the greatest weight, and each price value gets a smaller weight as Duff & Phelps will count backward in the series.

On the other hand, an EWMA model applies weighting factors which decrease, never reaching zero. EWMA also assigns a weighting factor to each value in the data series according to its age. Here as well the most recent data gets the greatest weight, and each price value will get a smaller weight as Duff & Phelps goes back in the series chronologically. The weight of each data point decreases exponentially, hence the name.

According to this, the EWMA model corresponds to the following:

$$\sigma_n^2(\text{EWMA}) = l\sigma_n^2 + (l - l)\, u_{n-1}^2. \qquad \text{(Equation 14)}$$

There are two options in defining the one parameter: assigning a consensus of the experts, which in this case would be around 95%, or maximising the verisimilitude of the probability function:

$$\sum(-L_n(\sigma_i^2) - u_1^2 / \sigma_i^2. \qquad \text{(Equation 15)}$$

From this point, it is important to forecast every data in order to conclude in forecasted volatilities. For this purpose, Duff & Phelps will develop a regression with a degree of delay of the log of returns. This regression concludes in a = 0.00115 and b = 0.38189.

Figure 10: Exponentially weighted moving average

				EWMA				
	Price (USD)	Ln (Rt)	Ln (Rt-1)	Regr. R*	Weight	Variance	Likelihood Probaility function	Volatility
Jan-05	31.94							
Feb-05	32.84	0.03						
Mar-05	38.11	0.15	0.03	0.01176	0.00%	0.08%	(21.56)	2.78%
Apr-05	39.84	0.04	0.15	0.05802	0.00%	0.18%	5.23	4.29%
Apr-17	52.31	0.01	(0.06)	(0.02239)	5.42%	1.08%	4.51	10.38%
May-17	50.33	(0.04)	0.01	0.00643	5.70%	1.02%	4.44	10.12%
Jun-17	46.30	(0.08)	(0.04)	(0.01360)	6.00%	0.98%	3.92	9.90%
Jul-17		(0.02)	(0.08)	(0.03071)	5.70%	0.97%	4.59	9.83%
Aug-17		(0.08)	(0.02)	(0.00680)	5.42%	0.92%	4.02	9.59%
Sep-17		(0.08)	(0.08)	(0.02885)	5.14%	0.90%	3.98	9.51%
Oct-17		0.14	(0.08)	(0.02983)	4.89%	0.89%	2.44	9.45%
Sep-21		0.07	0.00	0.00198	0.49%	1.15%	4.07	10.72%
Oct-21		0.12	0.07	0.02705	0.46%	1.12%	3.14	10.56%
Nov-21		0.05	0.12	0.04807	0.44%	1.14%	4.28	10.65%
Dec-21		0.24	0.05	0.01931	0.42%	1.09%	(0.62)	10.44%
Probability function (S/Avg)							**675.33**	**9.19%**
Annual Volatility								**31.84%**

The application of the aforementioned processes and formulas conclude in a forecasted volatility as follows (Figure 11):

Figure 11: Average volatility

	2017	2018	2019	2020	2021
Avg. Monthly volatility	10.02%	9.86%	964.00%	8.90%	8.08%
Avg. Annual volatility	35.0%	30.7%	28.6%	31.2%	32.4%

And a forecasted price scenario is shown in Figure 12:

Figure 12: EWMA estimation

	2017	2018	2019	2020	2021
EWMA estimation	**50.8**	**58.0**	**63.5**	**61.5**	**80.9**

ARCH MODELS

Engle[22] is the author of a first approximation to conditional variance. In order to justify the development of these models, the author cites two situations that cannot be explained by ARIMA models and which frequently appear in time series of financial data: (i) empirical experience leads Duff & Phelps to contrast periods of wide variance, followed by others of smaller variance, and (ii) Engle sets forth the validity of these models to determine the criteria to either hold or sell financial assets. Bollerslev, in 1986, generalised the Generalized Autoregressive Conditional Heteroscedastic Models (GARCH) models.

More flexible models are the stochastic volatility models introduced by Harvey, Ruiz and Shephard (1994), and Jacquier and Polson and Rossi. These models replicate some of the typical properties of financial series, such as excess kurtosis, clustering of volatility periods or correlation in the squares of the series.[23]

In summary, when considering volatility as a stochastic process, Duff & Phelps will seek to fit a model that allows us to describe and analyse its current behaviour and, from this, its future behaviour. For the case of constant variance processes, the Box-Jenkins methodology will be used. However, this assumption is not sustainable in several areas of research, so alternatives should be considered. Within these alternatives, Duff & Phelps highlights the ARCH and GARCH models proposed by Engle and Bollerslev, respectively, as these models allow us to specify the behaviour of the variance. A large number of studies on volatility models have been published in recent decades.

Heteroscedasticity refers to data coming from distributions of probability with different variances. This is one of the most important and interesting aspects of oil price behaviour, where the randomness of the Brownian movement, together with the influence of innumerable macroeconomic effects, provokes the practical impossibility of making reliable forecasts, according to probabilistic distributions analysed in historical periods.

First, Duff & Phelps will select the data under analysis. In this case, the monthly Brent data from July 1988 to June 2017 will be used, and from there, Duff & Phelps will calculate the logarithm of the returns, which will be the initial data.

Data corresponding to the projected series will be estimated according to the aforementioned distribution. The variance defined for the long term will be taken, and for the mean, Duff & Phelps will select the average returns calculated in the historical series.

$$\text{Ln Rt} = \mu + \sigma LP\ \varepsilon t \sim N(0,1) \qquad \text{(Equation 16)}$$

From this data, the formula for the calculation of the variance, proposed by Engle, will be developed:

$$\sigma n^2 = \omega + \alpha\ et^2 + \beta\ \sigma n{-}1^2 \qquad \text{(Equation 17)}$$

- Ω: parameter added to the EWMA model, referring to mean reverting
- $\sigma^2 LP$: estimated long-term variance
- ω, β and a: adjustment parameters of the model, assuming that $\omega + \beta + a = 1$ and $\beta + a < 1$

The parameters a, β and l will be estimated simultaneously by maximising the logarithm of the likelihood function:

$$\text{Log L} = \Sigma_{i=0}^{n} \left(-LN(\sigma_i^2) - e_i^2 / \sigma_i^2\right) \qquad \text{(Equation 18)}$$

As per the earlier equations, the obtained parameters will be as follows:

- $\omega = 0.4\%$
- a = 44.8%
- $\beta = 49.8\%$

The application of these parameters to Equation 17 will allow us to estimate the volatility for the projected periods (see Figure 13).

Figure 13: Average volatility

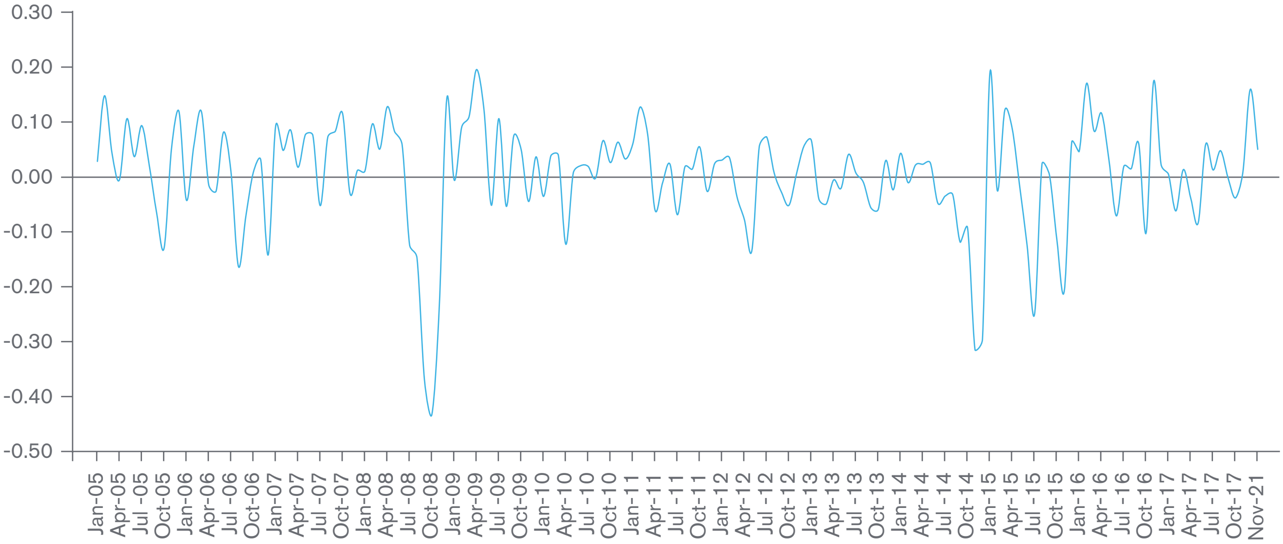
	2017	2018	2019	2020	2021
Avg. Monthly volatility	8.6%	10.6%	9.2%	7.9%	9.0%
Avg. Annual volatility	25.6%	33.8%	33.2%	28.3%	26.7%

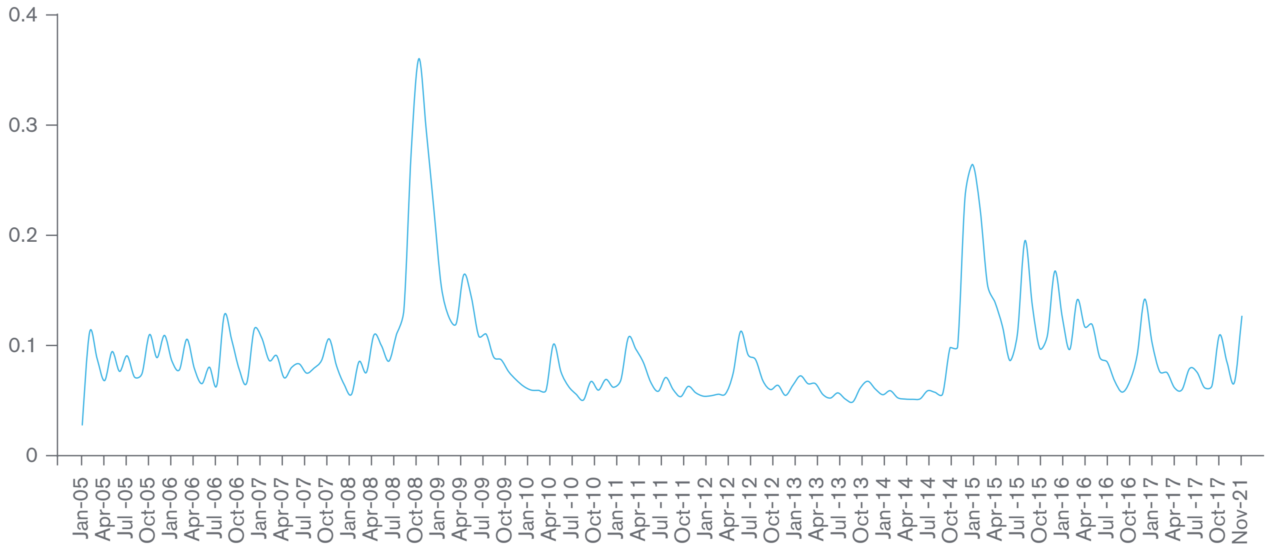
This will result in the following forecasted price scenario (Figure 14):

Figure 14: GARCH estimation

	2017	2018	2019	2020	2021
GARCH	**46.3**	**54.0**	**64.4**	**69.8**	**71.7**

Figure 15: Return of Brent prices and monthly volatility





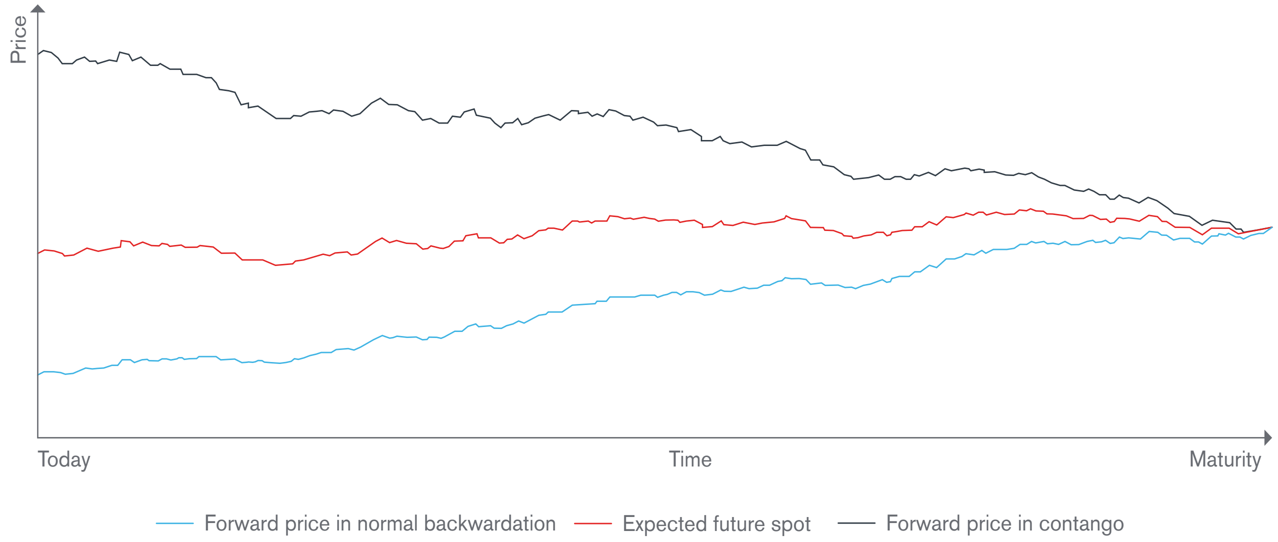
As it can be observed in Figure 15, monthly volatility is gradually approaching the volatility calculated for the long term (9.5%) in a mean-reverting movement.

Implied Volatility Analysis: Futures Market

As a necessary complement, Duff & Phelps will finalise the analysis of crude oil prices by incorporating a market opinion. This will be carried out through the futures quotation, under the premise that quotations, in normal conditions, would meet the spot price upon the delivery date.

In the event that this premise does not exist, the agent in charge of delivering the product will buy it in the market, taking profits or losses for the difference. Therefore, the prices stipulated in futures contracts may be a reasonable estimate of crude oil prices (Figure 16).

Figure 16: Futures market



According to Figure 16, and regarding the futures market, the term contango or contango market describes the market situation of a product or financial asset in which the spot price (the spot market price) of the asset is lower than the future price. The market situation in contango conveys that the price of the good will remain stable or rise in the future. The opposite situation is known as backwardation.

In oil markets, contango situations have reflected expectations of future supply tensions and led to the accumulation of stockpiles by buyers in order to face the expectation of an increase in prices. A contango market is normal for non-perishable products that have significant cost of transportation and storage. These costs include storage and unreceived interest for the money invested in the asset.

In addition, futures, as a means of perception for investors of the immediate future, provide interesting information to collate estimates achieved through statistical methods.

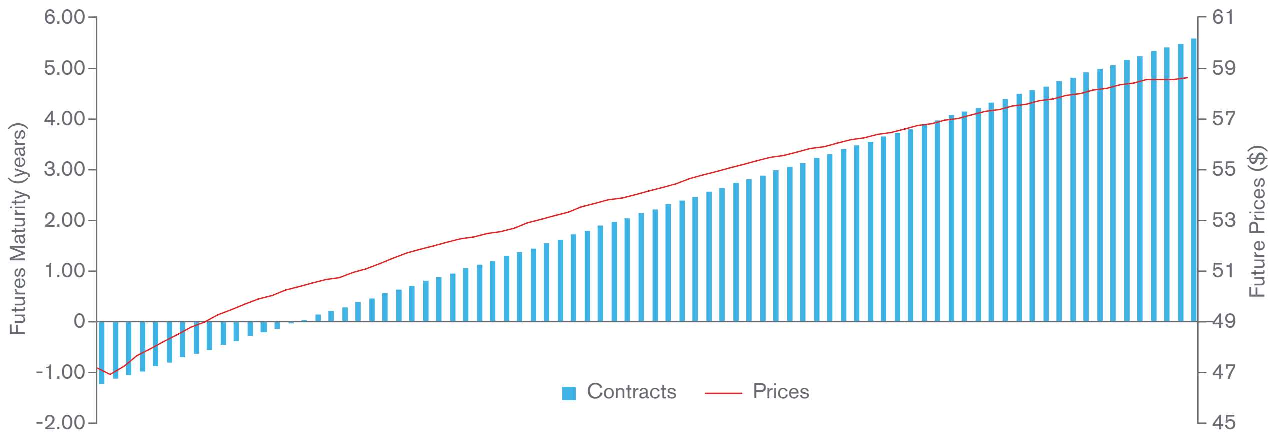
There are currently 82 Brent futures contract types traded on Intercontinental Exchange (ICE). The maturity period is obtained by subtracting the expiration date of the future, which is predetermined at the time of the issuance, from the current date. That is, a future that was issued on December 31, 2012, with a maturity date on December 31, 2018, will have a maturity period of five years at the end of 2013, four years at the end of 2014, three years at the end of 2015, two years at the end of 2016 and one year at the end of 2017.

Each Brent futures contract has its own expiration date. Basically, each month, one of the futures contracts, which are currently trading, reaches maturity, up to January 31, 2023.

Figure 17 shows the maturity period of each of the contracts. That is, what is left from today up to the day they are no longer listed. The red line shows the price of each of the futures to date. These two trends show a correlation of 97%; thus, the more distant the time of expiration, the greater the price.

The second part of Figure 17 shows the moment in which the contracts were issued – that is, the time left from the day they were issued until their expiration. The following table shows the time of issuance of the futures:

Figure 17: Futures maturity and duration period



It can be observed that the market is discounting the Brent price between US$45 and US$58 per barrel, which is absolutely consistent with the conclusions of the empirical study presented next.

Conclusion

According to the aforementioned methodologies, Duff & Phelps proposes different price scenarios, which are compiled in a single opinion calculated as an average.

The time span affected by this calculation will cover up to 2021, considering that, beyond five years, any forecast, in addition to being bold, is not reliable (Figure 18).

Figure 18: Conclusion

	2017	2018	2019	2020	2021
Wiener Process	56.8	49.8	47.5	68.9	51.7
Mean Reversion	51.7	56.8	61.2	64.5	66.7
Polynomial Regresion	55.7	64.1	72.9	81.4	89.0
ARIMA	51.7	54.2	57.0	59.9	62.8
EWMA	50.8	58.0	63.5	61.5	80.9
GARCH	46.3	54.0	64.4	69.8	71.7
Average	52.2	56.2	61.1	67.7	70.5
Standard Deviation	3.8	4.8	8.5	7.8	13.2

In reviewing the data obtained, it is inevitable to look back on historical information to determine whether the future replicates reality:

“OPEC Secretary-General Abdullah Badri has recently announced that oil could reach up to US$200 per barrel. Although this forecast now seems unreliable, there are reasons why it might be fulfilled”. – January 30, 2015

“In the short term, we are mainly affected by oil exporters, but in the medium and long term it harms everyone; importers, because of the destruction of the investments which fill the oil sector, the towers that by hundreds are being lost in the US for example, drilling, because it is no longer profitable to continue to exploit oil at such a low price, will cause a rebound in the medium term up to 200 dollars”. – Rafael Correa, President of Ecuador, February 22, 2016

“The lack of an adequate supply growth is evident. That the barrel is worth between 150-200 dollars is increasingly likely within six months and two years”. – Goldman Sachs, May 7, 2008

Endnotes

1 Michale P. Dooley, Peter M. Garber and David Folkerts-Landau described the new economic relationship between emerging economies and the United States as in Bretton Woods II.
2 Cheong 2009." Cheong, C.W. (2009) Modeling and forecasting crude markets using ARCH-type models. Energy policy, 2346-2355.
3 "On the Behavior of Commodity Prices".
4 In honor of Robert Brown.
5 Stochastic process with continuous time period, named thus in honor of Norbert Wiener. Frequently, these kinds of processes are called standard Brownian Motion.
6 "Random Walk or Mean Reverting? Empirical Evidence from the Crude Oil Market".
7 The martingale concept, in the probability theory, was introduced by Paul Pierre Lévy, and a large part of the original development was elaborated by Joseph Leo Doob.
8 Kiyoshi Itô (September 7, 1915–November 10, 2008) was a Japanese mathematician whose work is now called Itô calculus. The basic concept of this calculation is the integral of Itô, and the most important of the results is the Itô motto, which facilitates mathematical understanding of random events. His theory has many applications – for example, in financial mathematics.
9 In probability and statistics theories, a Markov process (named for the Russian mathematician Andrei Markov) – a random phenomenon dependent on the time for which the property of Markov is fulfilled or of a stochastic process without memory – is a phenomenon for which the conditional probability of the present, future and past state of the system are independent.
10 Established by the SEC as a value of the reserves that is a comparable parameter to all companies in the sector.
11 Duff & Phelps will take US$40 per barrel as a fair value because the US$25 per barrel achieved during the first months of 2016 is not representative or sustainable in the medium term.
12 This should be named as such because this characteristic is not accurate, or at least has not been maintained in time.
13 A variable whose values change over time in an uncertain manner is said to follow a stochastic process.
14 β in the regression.
15 Mean Reverting versus Random Walk in Oil and Natural Gas Prices". Hélyette Geman. 2007
16 Monographs by Juan Mascareñas regarding corporate finance. Stochastic processes". Code: ISSN: 1988-1878.
17 Higher-order concept when Duff & Phelps refers to oil prices.
18 Integrated autoregression model of mobile averages (ARIMA).
19 Autoregressive conditional heteroskedasticity (ARCH).
20 Weighted moving average (WMA).
21 Exponentially weighted moving average (EWMA).
22 1982.
23 "Forecasting Volatility in Financial Markets". Poon and Granger (2003), "A forecast comparison of volatility models: does anything beat a GARCH(1,1)?". Hansen and Lunde (2006), "Measurement of volatility in financial time series". Novales and Gracia (1993).

OUR SERVICES

Duff & Phelps Oil & Gas Solutions

Our Oil & Gas team underpins our success in a unique capability to translate former experience as consultancy and investment banking high professionals into a deep understanding of the industry, and particularly National Oil Companies, leveraging the fat that we have worked closely with most industry members for years to create solvent solutions in complex situations. This expertise allows us to be a valued interlocutor between both sides of the deal. Our services include:

Transaction Advisory

- Farm-in and farm-out analysis;
- Optimized cost allocation;
- Joint venture analysis;
- Contract due diligence; and
- Acquisition or vendor due diligence.

Valuation

- Valuation of blocks;
- Valuation of business;
- Valuation of working interest in a joint venture;
- Valuation in a migration context;
- Valuation of contracts;
- Valuation of tax schemes; and
- Valuation of common shares.

Mergers and Acquisitions

- Non-core business units divestment;
- Management Buyout and Management Buy-in for oil services companies;
- Development of negotiation frameworks; and
- Screening of potential investors looking for Oil & Gas opportunities.

Financial Advisory

- Loan convenant review;
- Risk modeling;
- Analysis of loan portfolio;
- Collateral analysis;
- Securitization; and
- Project finance.

Strategic Advisory

- Design of fiscal schemes;
- Advisory in the context of energy reforms;
- Expertise in privatization processes; and
- Advisory on transitions from fossil fuels to renewable energies.

The Duff & Phelps Difference

Duff & Phelps is the premier global valuation and corporate finance advisor.

Duff & Phelps has a worldwide Oil & Gas team with extensive experience in accounting, assets valuation, financial valuation, M&A and strategic advisory services;

The Oil & Gas team has been involved in the major Oil & Gas transactions;

The Oil & Gas team is made up of former investment banking professionals and Big Four consultants and engineers, all of them specialized in the industry;

Excellent access to industry experts, banks and investors;

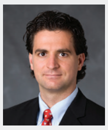
Leading-edge technical expertise in valuation and simulation, transaction strategies and financial impact analysis;

Extensive track record advising on complex valuation, M&A and internal control assignments.

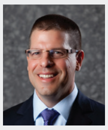
DUFF & PHELPS

CONTACTS

Fernando Ramírez de Verger, Ph.D.
Managing Director, Madrid
Global Valuation Advisory Services Oil & Gas Leader
fernando.ramirezdeverger@duffandphelps.com



David Herr
Managing Director, Philadelphia
Global Energy and Mining Industry Leader
david.herr@duffandphelps.com



Jim Rebello
Managing Director, Houston
Global Head of Energy M&A
james.rebello@duffandphelps.com

Content based on the registered doctoral thesis – 16/2017/1859 by Fernando Ramírez de Verger, Ph.D.

Study of the Financiarization of the Oil through Price Forecasting Inferential Models and Analysis of Its Impact on the Market Uncertainty through Binomial Lattices

In accordance with the stipulations of the Intellectual Property Law (Royal Legislative Decree 1/1996 of 12 April), intellectual property rights are registered as Register Number 16/2017/1859.

About Duff & Phelps
Duff & Phelps is the premier global valuation and corporate finance advisor with expertise in complex valuation, disputes and investigations, M&A, real estate, restructuring, and compliance and regulatory consulting. The firm's more than 2,000 employees serve a diverse range of clients from offices around the world.

For more information, visit www.duffandphelps.com

© 2017 Duff & Phelps, LLC. All rights reserved. DP170177

M&A advisory, capital raising and secondary market advisory services in the United States are provided by Duff & Phelps Securities, LLC. Member FINRA/SIPC. Pagemill Partners is a Division of Duff & Phelps Securities, LLC. M&A advisory and capital raising services in Canada are provided by Duff & Phelps Securities Canada Ltd., a registered Exempt Market Dealer. M&A advisory, capital raising and secondary market advisory services in the United Kingdom and across Europe are provided by Duff & Phelps Securities Ltd. (DPSL), which is authorized and regulated by the Financial Conduct Authority. In Germany M&A advisory and capital raising services are also provided by Duff & Phelps GmbH, which is a Tied Agent of DPSL. Valuation Advisory Services in India are provided by Duff & Phelps India Private Limited under a category 1 merchant banker license issued by the Securities and Exchange Board of India.