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The Size Effect Continues To Be Relevant When Estimating the Cost of Capital

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*In this paper, I will review the size effect, potential reasons why one observes the size effect, and correct common misconceptions and address criticisms of the Size Premia (*SP*). Specifically, we demonstrate that the size premium critique by Cliff Ang[2] is not warranted and that the alternative methodology proposed by that author is misleading and cannot be considered as an alternative to the Duff & Phelps'* SP. *Subsequently, we will highlight some methodological issues with his proposed alternative. The methodology the author is proposing is picking up the statistical errors that he was set to avoid by proposing the same methodology. I will discuss other criticisms we have encountered. Finally, I will provide some practical guidance on applying* SP.

Introduction

Valuation professionals frequently incorporate Size Premia (*SP*) in developing the cost of equity capital using the modified capital asset pricing model (MCAPM) to estimate the correct cost of capital for smaller firms.[3] Shannon Pratt and this author have previously addressed many of the issues surrounding the size effect in the *Cost of Capital: Applications and Examples*, 5th ed.[4], and we will be writing more extensively on the topic in the next edition. This author recently wrote on a paper on *SP* to correct some misconceptions about *SP* and to show that—contrary to claims made by critics—data covering recent periods supports continued use of *SP*.[5]

The debate that has been raging in the academic sphere about the nature of this adjustment relates to its nature. Is it a risk factor that investors should be compensated for? Or is it related to firm characteristics? The jury is still out on this debate, but the need for an adjustment for the cost of capital relative to size is a something that professionals need to consider given the limitations of CAPM and the market portfolio in explaining equity returns.

Size Effect—Brief History

The size effect is based on numerous empirical studies that show that companies of smaller size are associated with greater risk and, therefore, have a greater cost of capital. In other words, an observable (negative) relationship occurs between size and realized equity returns—as size *decreases*, returns tend to *increase*, and vice versa.

Traditionally, researchers have used market value of equity (market capitalization, or simply "market cap") as a measure of size in conducting historical rate of return studies. However, market cap is not the only measure of size that can be used to predict returns, nor is it necessarily the best measure of size. In fact, the use of market cap as a measure of size is the cause of much of the confusion about the size effect.

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[1] I thank Anas Aboulamer, PhD, Duff & Phelps, for assistance with this paper.

[2] Clifford S. Ang, "Why We Should Not Add a Size Premium to the CAPM Cost of Equity," (June 27, 2016), accessed at http://quickreadbuzz.com/2017/02/15/shouldnt-add-size-premium-capm-cost-equity/, September 13, 2018.

[3] Shannon P. Pratt and Roger J. Grabowski, "Capital Asset Pricing Model," Chap. 10 of *Cost of Capital: Applications and Examples*. 5th ed. (Hoboken, New Jersey: John Wiley & Sons, Inc., 2014).

[4] Shannon P. Pratt and Roger J. Grabowski, "Criticisms of the Size Effect," Chap. 15 of *Cost of Capital: Applications and Examples*. 5th ed. (Hoboken, New Jersey: John Wiley & Sons, Inc., 2014).

[5] Roger J. Grabowski, "The Size Effect—It is Still Relevant," *Business Valuation Review* 35 (2016): 62–71; data is presented from 1990 through 2014.

One of the first characteristics that researchers analyzed was returns on large-market-capitalization (large-cap) companies versus returns on small-market-capitalization (small-cap) companies because the data to calculate market capitalization were available in the Center for Research in Security Prices (CRSP) database. For example, a 1981 study by Rolf Banz examined the returns of New York Stock Exchange (NYSE) small-cap companies compared to the returns of NYSE large-cap companies over the period 1926 to 1975. Banz observed that the returns of small-cap companies were *greater* than the returns for large-cap companies.[6]

Roger Ibbotson and Rex Sinquefeld began publishing summaries of data that demonstrated the size effect (later incorporated in the annual *Stocks, Bonds, Bills and Inflation* [*SBBI*]) based on analyzing the CRSP data and market cap as the measure of size.[7] Originally they published a small stock premium calculated as the simple difference in small-cap returns versus large-cap returns.[8] However, relying on the simple difference in small-cap returns versus large-cap returns is problematic because

> in doing so one assumes that the company being valued has the same systematic risk (or beta) as the portfolio of small stocks used in the calculation of the size premium.[9]

In other words, *SP* are now most often measured by removing the portion of observed excess return that is attributable to the CAPM beta, leaving only the size effect's contribution to excess return. For each portfolio constructed from the CRSP database measuring size by market cap, *SP* are calculated as follows:

$$\text{Size premium} = \text{Realized return} - \text{Estimated return},$$

where the realized return equals the historical return in excess of the risk-free rate (calculated as the realized long-term arithmetic mean return of the subject portfolio of stocks minus the realized long-term arithmetic return of the risk-free rate) and the estimated return equals the return expected from CAPM (calculated as beta for the subject portfolio of stocks multiplied by the realized equity risk premium, the expected return on the market portfolio of stocks in excess of the risk-free rate).

This author still reads references to "small stock premium" when the writer means *SP* and vice versa. To avoid confusion, it is important to accurately define the relationship one is discussing.

While many critiques of the size effect focus on the small stock premium with size measured by market capitalization, we shall examine the beta-adjusted size premia now published as the CRSP Decile Size Premia.[10]

First, why do we observe the empirical CRSP Decile Size Premia? For example, is the size effect simply the result of not estimating beta correctly? That is, if we had a better way of estimating beta the CAPM would we not observe the *SP*? Or, are there simply market anomalies that cause the size effect to appear? Is size a proxy for one or more other factors correlated with size? Should one directly use these factors rather than size to measure risk? Is the size effect hidden because of unexpected events?

Possible Explanations for the Size Effect

SP are based on empirical observations. The observed returns of small-cap stocks adjusted for beta risk in excess of returns of large-cap stocks calls into question whether the *textbook* CAPM fully explains stock returns. If CAPM were fully explaining the returns on stocks, then why do we find a size effect after adjusting for beta? This question has spawned a large body of research.

Small companies are believed to have greater required rates of return than do large companies because small companies are inherently riskier. It is not clear, however, whether this is attributable to size itself or to other factors closely related to or correlated with size. Banz's insight in his 1981 article remains as pertinent today as it was thirty-seven years ago:

> It is not known whether size [as measured by market capitalization] per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.[11]

Practitioners know that small firms measured in terms of fundamental size measures such as assets or net income have risk characteristics that differ from those of large firms. For example, potential competitors can more easily enter the "real" market (the market for the goods and/or services offered to customers) of the small firm and take the value that the small firm has built. Large companies

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[6] Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981):3–18. This paper is often cited as the first comprehensive study of the size effect.

[7] Later the *SBBI Yearbook* was replaced by the *SBBI Valuation Yearbook* (which was published by Morningstar until 2013). The data series was more recently published in the Duff & Phelps *Valuation Handbook – U.S. Guide to Cost of Capital* from 2014 through 2016. The data series is now available via the online Duff & Phelps Cost of Capital Navigator platform. *See* dpcostofcapital.com and view a video case study to see the functionality and capabilities of the Cost of Capital Navigator.

[8] The excess returns to small cap stocks data series continues to be available via the on-line Cost of Capital Navigator under the title "The CRSP Decile Size Premia Studies."

[9] Author, *2012 SBBI Valuation Yearbook* (City: Publisher, 2012), 28.

[10] Published in 2014 through 2017 in the annual *Valuation Handbook—U.S. Guide to Cost of Capital* and now available through the online Cost of Capital Navigator online platform.

[11] Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981):3–18.

have more resources to better adjust to competition and avoid distress in economic slowdowns. Small firms undertake less research and development and spend less on advertising than large firms do, giving them less control over product demand and potential competition. Small firms have fewer resources to fend off competition and redirect themselves after changes in the market occur.[12]

Smaller firms often have fewer analysts following them and less information available about them. Smaller firms may have less access to capital, thinner management depth, a greater dependency on a few large customers, and their stocks may be less liquid than the stocks of their larger counterparts. One study found that analysts and investors have difficulty evaluating small, little-known companies and estimating traditional quantitative risk measures for them. This ambiguity adds to the risk of investment and increases the return required to attract investors.[13]

The characteristics of smaller firms generally cause the rate of return that investors expect when investing in stocks of small companies to be greater than the rate of return expected when investing in stocks of large companies.

Several authors have investigated problems with estimating beta, a forward-looking concept, using historical returns over look-back periods. The most common method used in estimating beta is the ordinary least squares (OLS) regression of historical returns over a look-back period. If the true beta is underestimated, *SP* will be observed, and the cost of equity capital estimated using the textbook CAPM will be underestimated.[14] *SP* can be seen as a correction for this underestimation.

For example, papers investigated the problem with underestimating betas for "troubled" firms that tend to populate the smaller deciles where size is measured by the market value of equity.[15] As the market value of equity gets bid down for a troubled company, its stock may trade like a call option.

This suggests that *SP* may be overestimated in CRSP subdecile 10z, for example, which is populated with the smallest companies as measured by market cap. The CRSP Decile Size Premia include all companies with no exclusion of speculative (e.g., start-up) or distressed companies whose market cap is small because of being speculative or distressed. Some critics have held that the inclusion of speculative or distressed companies in the database is a basis for criticism of the size effect.[16] We will return to a discussion of subdecile 10z later.

Size Effect—More Research

In 1990, this author, who regularly applied *SP* when estimating the cost of equity capital using the MCAPM, was confronted with criticisms of the size effect and began closely studying the relationship between company size and stock returns. This research focused on whether stock returns were predicted by measures of size other than market cap and whether stock returns were predicted by fundamental risk measures based on accounting data. We found that as size decreases, or risk increases (as measured by fundamental accounting data), returns tend to increase (and vice versa). Thereafter, we published a series of articles reporting our findings, culminating with a seminal 1996 article and a subsequent article in 1999 that together serve as the foundation of the Risk Premium Report studies.[17]

These studies differ from some of the academic research in the way the portfolios are constructed. The basic methodology is akin to that used by valuation professionals when identifying guideline public companies when valuing a nonpublicly traded business. Valuation professionals begin their investigation by searching for appropriate guideline public companies to include in their estimation of beta and their market approach analysis by examining the characteristics of potential guideline public companies and comparing those characteristics to those of the subject business.

The valuation professional strives to identify the hypothetical "as if publicly traded" market value of the subject company by comparing its metrics to those of publicly traded companies with comparable risk and expected return characteristics as the subject company. For example, if the subject company is an established small company, the most appropriate guideline companies are those established publicly traded companies that are in the same industry but are also small. Likewise, if the subject company is not highly levered and is profitable, the most appropriate guideline pubic companies in its industry are those that are not highly levered and are profitable. It is

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[12] See, for example, M. S. Long and J. Zhang, "Growth Options, Unwritten Call Discounts and Valuing Small Firms," EFA 2004 Maastricht Meetings Paper no. 4057, March 2004.

[13] R. Olsen and G. Troughton, "Are Risk Premium Anomalies Caused by Ambiguity?" *Financial Analysts Journal* (March–April 2000):24–31.

[14] Sabine Elmiger, "CAPM-Anomalies: Quantitative Puzzles," *Econ Theory* (June 2018). http://doi.org/10.1007/s00199-018-1137-5.

[15] Carlos A. Mello-e-Souza, "Bankruptcy Happens: A Study of the Mechanics of Distressed Driven CAPM Anomalies," Working paper, January 25, 2002; and "Limited Liability, the CAPM and Speculative Grade Firms: A Monte Carlo Experiment," Working paper, August 18, 2004.

[16] Jonathan B. Berk, "A Critique of Size Related Anomalies," *Review of Financial Studies* 8 (Summer 1995):225–286.

[17] Roger J. Grabowski and David King, "New Evidence on Size Effects and Equity Returns," *Business Valuation Review* 15 (September 1996, revised March 2000):103–115; Roger J. Grabowski and David King, "New Evidence on Equity Returns and Company Risk," *Business Valuation Review* 18 (September 1999, revised March 2000):112–130.

easy to understand in applying the market approach (i.e., applying multiples derived from guideline public companies to the subject company). Even then, some academics erroneously claim that valuation professionals should apply an average multiple drawn from all companies in an industry whether they would be considered guideline companies or not. This faulty line of reasoning was countered in recent empirical research.[18]

The same logic follows in examining the returns of similar public companies to develop a discount rate appropriate for the subject company. That is, the historic returns of companies with comparable characteristics as those of the subject company should be used as evidence of the likely expected returns for the subject company. That logic guided us in constructing the portfolios reported in the Risk Premium Report studies.

The Risk Premium Report studies screen out speculative start-ups, distressed (i.e., bankrupt) companies, and other financial high-risk companies. We are examining the returns of relatively high-quality firms.[19] This methodology was chosen to counter the criticism of the size effect by some that the *SP* is a function of the high rates of return for speculative companies and distressed companies in the data set.

Financial services companies are also excluded from the analysis because the regulated nature of banks and insurance companies causes their underlying characteristics to differ from those of non-regulated companies.[20]

Risk Premium Report–Size Studies[21]

The Risk Premium Report–Size Studies report on size premia where size is measured in eight different measures: Market capitalization; Book value of equity; Five-year average net income; Market value of invested capital (MVIC); Total assets; Five-year average EBITDA; Sales; Number of employees.

We have several reasons for using alternative measures of size (in addition to the market cap used in the CRSP Decile Size Study).

First, financial literature indicates that a bias may be introduced when ranking companies by market value because a company's market capitalization may be affected by characteristics of the company other than size. In other words, some companies might be small (as measured by market cap) because they are risky (high discount rate), rather than risky because they are small (small assets or small income).[22]

One simple example could be a company with a large asset base but a small market capitalization as a result of high leverage or depressed earnings. Another example could be a company with large sales or operating income but a small market capitalization attributable to being highly leveraged.

Second, market cap may be an imperfect measure of the risk of a company's operations.

Third, using alternative measures of size may have the practical benefit of removing the need to first make a guesstimate of size (i.e., the hypothetical market cap of the subject company) in order to know which portfolio's premium to use (this issue is commonly referred to as the "circularity" issue). When you are valuing a nonpublicly traded company, you are trying to determine an estimate of as if public traded market value. If you need to make a guesstimate of the subject company's market cap first in order to know which size premia to use, the circularity problem is introduced. While market cap for the nonpublicly traded company is not available, other size measures, such as assets, net income or sales, are generally available.

Returns each year are measured as the equal weighted average return for the companies comprising the size ranked portfolio. The valuation professional is not building investment portfolios but rather, we are determining the return for the typical company with certain size characteristics over time.[23]

For illustrative purposes, we focus on the results of the Size Study for four measures of size based on returns for 1981 through 2016 and 1990 through 2016[24] for the portfolios comprised of the smallest companies (as measured by the respective size measure) (Portfolios 21 through 25) (see Exhibits 1, 2, 3, and 4). The size premia (observed premium over CAPM) is observed for both periods, 1981–2016 and 1990–2016 (see Premium over CAPM columns).

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[18] Friedrich Christian Rose Sommer and Arnt Wohrmann, "Negative Value Indicators in Relative Valuation—An Empirical Perspective," *Journal of Business Valuation and Economic Loss Analysis* 9 (2014):23–54.

[19] Thus this analysis can be thought of as consistent with the findings of Clifford S. Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse Heje Pedersen, "Size Matters, If You Control Your Junk," *Journal of Financial Economics* 129 (2018):479–509.

[20] Because financial services companies are excluded from the base set of companies used to develop the analyses presented in the Risk Premium Report studies, the data published should not be used to estimate cost of equity for financial services companies (i.e., companies with an SIC Code that begins with 6).

[21] See Chapter 7, "The CRSP Decile Size Premia Studies and the Risk Premium Report Studies—A Comparison"; Chapter 9, "Risk Premium Report Exhibits—General Information"; and Chapter 10, "Risk Premium Reports—Examples," available through the online Cost of Capital Navigator platform.

[22] Jonathan B. Berk, "A Critique of Size Related Anomalies," *Review of Financial Studies* 8 (Summer 1995):225–286.

[23] This is comparable to the S&P Equal Weighted Index.

[24] The exhibits presented herein are based on data extracted from the data used to prepare Risk Premium Report Exhibit B that appeared in the *2017 Valuation Handbook—Guide to Cost of Capital* for the period 1963 to 2016. Of course, in developing cost of capital estimates, one should use data for the period ending prior to the valuation date.

Exhibit 1
Companies Ranked by Size Measured by Market Value of Equity: Size Premia for Companies Ranked by Market Value of Equity

Portfolio Ranking by Size	Average MVE (in $Millions)*	Premiums of CAPM 1981–2016	Premiums of CAPM 1990–2016
21	1,023	2.82%	4.27%
22	731	3.38%	4.83%
23	532	2.36%	3.81%
24	370	5.61%	7.06%
25	121	7.99%	9.43%

* Average total market value of equity (MVE) in 2015 for companies comprising each portfolio
CAPM = capital asset pricing model.

We show these specific periods to counter the criticism that *SP* have disappeared in the post-Banz periods.

While the change in *SP* portfolio to portfolio are not uniformly monotonic, we are presenting empirical results. We make no claim that there is an underlying theory. Rather, *SP* are a correction to a theory shown to be fraught with problems.

In summary, given this recent evidence, one can conclude that the size effect can still be used today by valuation professionals.

Is the Size Premium a Proxy for Other Characteristics?

Size and fundamental risk of small companies are related. Exhibits 5, 6, 7, and 8 display select data for the same periods as shown in Exhibits 1 to 4 above. We display the data for the portfolios comprising the largest firms as measured by the respective measure of size,

Exhibit 2
Companies Ranked by Size Measured by Five-Year Average Net Income: Size Premia for Companies Ranked by Five-Year Average Net Income

Portfolio Ranking by Size	Net Income (in $Millions)*	Premiums of CAPM 1981–2016	Premiums of CAPM 1990–2016
21	42	1.68%	3.13%
22	34	3.01%	4.95%
23	24	4.17%	5.62%
24	15	4.41%	5.86%
25	5	6.18%	8.05%

* Five-year average prior to 2016 for companies comprising each portfolio
CAPM = capital asset pricing model.

Exhibit 3
Companies Ranked by Size Measured by Total Assets: Size Premia for Companies Ranked by Total Assets

Portfolio Ranking by Size	Average Total Assets (in $Millions)*	Premiums of CAPM 1981–2016	Premiums of CAPM 1990–2016
21	1,069	2.15%	3.59%
22	801	3.15%	4.60%
23	600	3.44%	4.89%
24	429	3.86%	5.31%
25	161	6.43%	7.87%

* Average total assets in 2015 for companies comprising each portfolio
CAPM = capital asset pricing model.

Portfolios 1, 2, and 3, and the data for the portfolios comprising the smallest firms as measured by the respective measure of size, Portfolios 21 through 25.

We make the following observations:

- The increase in *SP* as size decreases is not the result of significantly different amounts of debt among the companies comprising the portfolios (see Avg Debt to MVIC columns); recall that the high financial risk companies have been excluded in the basic Risk Premium Report analysis.
- Business risks as measured by the unlevered asset beta (i.e., greater asset beta indicates greater business risk) generally increase as size decreases (see Average Unlevered Beta column).
- Business risks as measured by the average operating margin (i.e., a lower average operating margin indicates greater business risk) generally increase as size decreases (see Average Operating Margin column).

Exhibit 4
Companies Ranked by Size Measured by Five-Year Average EBITDA: Size Premia for Companies Ranked by Five-Year Average EBITDA

Portfolio Ranking by Size	Five-year Average EBITDA (in $Millions)*	Premiums of CAPM 1981–2016	Premiums of CAPM 1990–2016
21	125	3.41%	4.85%
22	94	3.12%	4.57%
23	74	4.68%	6.13%
24	51	3.30%	4.74%
25	17	5.99%	7.43%

* Five-year average prior to 2016 for companies comprising each portfolio
CAPM = capital asset pricing model; EBITDA = earnings before interest, taxes, appreciation, and amortization.

Exhibit 5

Companies Ranked by Size Measured by Market Value of Equity: Fundamental Risk Data for Companies Ranked by Market Value of Equity

Portfolio Ranking by Size	MVE (in Millions)*	Fundamental Risk Data		Avg Operating Margin	Avg CV (Operating Margin)
		Avg Debt/MVIC	Unlevered Beta		
1	238,299	13.13%	0.68	19.09%	9.35%
2	60,613	16.50%	0.83	14.74%	12.01%
3	35,630	18.49%	0.72	14.72%	11.96%
21	1,023	18.97%	0.93	8.84%	24.96%
22	731	19.29%	0.98	8.73%	26.73%
23	532	18.81%	0.98	8.00%	29.52%
24	370	19.02%	1.01	7.99%	33.00%
25	121	21.94%	0.94	6.27%	47.18%

* Average in 2015 for companies comprising each portfolio
MVIC = debt plus market value of equity; CV = coefficient of variation.

Exhibit 6

Companies Ranked by Size Measured by Five-year Average Net Income: Fundamental Risk Data for Companies Ranked by Five-Year Average Net Income

Portfolio Ranking by Size	Net Income (in Millions)*	Fundamental Risk Data		Avg Operating Margin	Avg CV (Operating Margin)
		Avg Debt/MVIC	Unlevered Beta		
1	10,101	17.38%	0.63	18.74%	9.51%
2	2,747	21.58%	0.71	15.38%	11.08%
3	1,735	23.54%	0.63	14.89%	11.44%
21	42	20.13%	0.98	9.41%	24.63%
22	34	19.82%	0.95	8.83%	25.26%
23	24	17.32%	1.02	8.75%	28.81%
24	15	18.61%	0.98	7.97%	33.87%
25	5	19.47%	0.99	5.97%	54.51%

* Five-year average prior to 2016 for companies comprising each portfolio
MVIC = debt plus market value of equity; CV = coefficient of variation.

- Business risks as measured by the variability of operating margin over the prior five years (i.e., a higher coefficient of variation of operating margin indicates greater risk) generally increase as size decreases (see Average CV [Operating Margin] column).

Liquidity affects the cost of capital.[25] For this purpose, *liquidity* refers to the speed at which a large quantity of a security can be traded with a minimal impact on the price and with the lowest transaction costs. Stocks of small companies generally do not have the same level of liquidity as large-company stocks. This is likely a function of the mix of shareholders and underlying risk characteristics. Many institutional investors do not own stocks in small companies because they have too much money to invest relative to the size of these companies. Were they to invest as little as 1% of their available funds in a small company, they likely would control the company. Institutional investors generally want sufficient liquidity to move into and out of their positions in a single firm without disrupting the market. Therefore, one does not see the breadth of investors investing in small-cap stocks, as one sees in large-cap stocks.

Further, small companies are followed by only a small number of analysts, if any at all. This makes it more difficult for investors to acquire information on and evaluate small firms.

[25] See, for example, Roger G. Ibbotson and Daniel Y.-J. Kim, "Risk and Return within the Stock Market: What Works Best?" Working paper, January 8, 2016. Accessed at http://www.zebracapital.com.

Exhibit 7
Companies Ranked by Size Measured by Total Assets: Fundamental Risk Data for Companies Ranked by Total Assets

Portfolio Ranking by Size	Total Assets (in Millions)*	Fundamental Risk Data		Avg Operating Margin	Avg CV (Operating Margin)
		Avg Debt/MVIC	Unlevered Beta		
1	161,117	26.16%	0.63	15.32%	13.77%
2	51,936	30.22%	0.57	15.66%	13.43%
3	35,110	26.02%	0.69	13.09%	12.95%
21	1,069	18.60%	0.97	9.04%	23.18%
22	801	17.55%	1.01	9.03%	25.69%
23	600	16.53%	0.99	8.44%	28.53%
24	429	16.23%	1.02	8.21%	30.74%
25	161	14.97%	0.99	7.33%	43.99%

* Average total assets in 2015 for companies comprising each portfolio
MVIC = debt plus market value of equity; CV = coefficient of variation.

Are *SP* observed for smaller companies (after adjusted for differences in beta) the result of difference in size or differences in liquidity? Differences in liquidity certainly impact the observed returns in publicly traded stocks but if one is estimating the cost of capital for a small, nonpublic business, the analyst has no observations as to the liquidity that company's stock might have were it public. We can only estimate that the liquidity of that stock would be similar of other publicly traded stocks of companies of similar size.

Fundamental risk may be creating the liquidity effect. That is, the greater underlying risks of small companies relative to those of larger companies may cause investors to shy away from small companies, reducing their liquidity. Thus, reduced liquidity may also be a coincident indicator of fundamental risk.

Criticisms of Ang

We will now provide a response to Ang's criticisms advocating that the CRSP Decile *SP* calculated by Duff & Phelps is erroneous. We will explain how his logic is flawed and present concrete evidence to that effect.

Criticism: Size Premium is an error is a statistical sense

Ang considers *SP* nothing but an excess of return that is not captured by the CAPM and added back labeled a Size Premium. The author refers to the size premium as

Exhibit 8
Companies Ranked by Size Measured by Five-Year Average EBITDA: Fundamental Risk Data for Companies Ranked by Five-Year Average EBITDA

Portfolio Ranking by Size	EBITDA (in Millions)*	Fundamental Risk Data		Avg Operating Margin	Avg CV (Operating Margin)
		Avg Debt/MVIC	Unlevered Beta		
1	22,452	20.52%	0.62	17.43%	10.90%
2	6,905	26.38%	0.64	15.19%	13.81%
3	4,343	26.51%	0.66	14.83%	11.29%
21	125	19.08%	0.95	9.71%	22.79%
22	94	18.58%	1	9.23%	24.42%
23	74	18.06%	0.98	8.41%	28.49%
24	51	17.30%	0.99	8.19%	32.27%
25	17	16.22%	1.01	6.80%	47.60%

* Five-year average prior to 2016 for companies comprising each portfolio
EBITDA = earnings before interest, taxes, depreciation, and amortization; MVIC = debt plus market value of equity; CV = coefficient of variation.

an "error" in a statistical sense. However, none of the academic papers throughout the last three decades have qualified the *SP* as a statistical error. The harshest criticism of *SP* amounted to its inability to represent an actual risk but merely related to firm characteristics, hence the stability of the premium over time. This criticism started with Daniel and Titman[26] and continued as the major concern about adopting Professors Eugene Fama and Kenneth French's (F-F) SMB (Small Minus Big) as a factor for pricing assets.

The author refers to data mining, which is not a statistical error per se. Data mining is finding an empirical relationship that could not hold in all data samples. The fact that *SP* is possibly obscured over a certain period by other factors does not negate its existence. The question that academics and other researchers are asking is why it exists over a period and not another, and what economic logic behind it. Asness et al. argue that all the challenges to *SP* disappear and it becomes stable over time when controlling for firm quality in twenty-four international equity markets and thirty industries.[27]

Another explanation for the use of the phrase "statistical error" by the author is his concern about the possibility that *SP* captures idiosyncratic components of individual firms that do not have any systematic pricing capability. Calculations of the *SP* requires the use of a portfolio of firms precisely to avoid being hijacked by the idiosyncratic components of individual members of the portfolio. Hence, when *SP* is calculated, it would refer to the additional return required by the holder of a typical (average) firm in that portfolio typical size (or decile), not a specific individual firm.

Duff & Phelps' calculations of CSRP Decile *SP* are based on large portfolios of firms diversifying all idiosyncratic risk of individual firms. The number of companies in the size portfolios is great enough to diversify the nonsystematic risk. For example, the historical average number of stocks in the lowest CRSP decile is around 1,300 firms with a low of 50 in the 1920s and a high of 3,575 in 1997. The historical average number of stocks in the highest CRSP decile is 124 with a low of 50 in the 1920s and a high of 124 stocks in 2000. These high numbers largely exceed the number of stocks required to diversify idiosyncratic risk in a portfolio of around 40.[28]

The expected return of a portfolio using MCAPM is:

$$r_p = r_f + \beta(r_m - r_f) + SP + \varepsilon_p$$

As the number of stock in the portfolio p increases, the value of ε_p goes to zero. Because the number of stocks in all CRSP deciles is more than fifty, we can confidently say that *SP* is:

$$SP = r_p - \Big(r_f + \beta(r_m - r_f)\Big)$$

and it does not contain any idiosyncratic components of individual stocks.

Criticism: Duff & Phelps Size Premia are biased

Ang argues that *SP* does not match how valuation professionals calculate the betas. His argument assumes that size factor is conditional on short term movement of stock returns and should be adjusted to reflect recent information. This reasoning could be right if we knew exactly what "risk" the size factor represents. In the case of beta (market risk) for example, a time interval of five years is used typically for the look-back period over which beta is estimated to reflect the market risk profile of the firm during a typical economic cycle. The five-year look-back is a rule of thumb with no scientific evidence to justify this length. It was thought that beta was unconditional (i.e., time invariant) until Jagannathan and Wang (1996) proved that it needs to be time variant to work.[29] In the case of *SP*, this condition is not satisfied at least for the time being. Hence, the most conservative way to proceed is to take a long-term average of the performance of the size portfolios.

How is the "Practitioner Consistent Size Premium" different?

Ang argues for a new methodology to calculate the *SP* dubbed "Practitioner Consistent Size Premium" (*PCSP*). His methodology purports to remove the bias and what he perceives as errors in estimating the *SP*.

The author methodology does not provide an original way of calculating the premium and it is essentially the same as what was found in *Cost of Capital: Applications and Examples* 5th ed. and in the Duff & Phelps *Valuation Handbook – U.S. Guide to Cost of Capital*.[30] His methodology is a mere change in the length of the

[26] Kent Daniel, Mark Grinblatt, Sheridan Titman, and Russ Wermers, "Measuring Mutual Fund Performance with Characteristic-Based Benchmarks," *The Journal of Finance* 52 (1997):1035–1058.

[27] Clifford S. Asness, Andrea Frazzini, Ronen Israel, Tobias J. Moskowitz, and Lasse Heje Pedersen, "Size Matters, If You Control Your Junk," *Journal of Financial Economics* 129 (2018):479–509.

[28] Meir Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (1987):353–363.

[29] Ravi Jagannathan, and Zhenyu Wang, "The Conditional CAPM and the Cross-Section of Expected Returns." *The Journal of Finance* 51 (1996):3–53.

[30] Published 2014 through 2017 in the annual *Valuation Handbook – U.S. Guide to Cost of Capital* and now available through the online Cost of Capital Navigator online platform.

look-back period used to estimate the betas of the portfolios (three years) and the period to calculate market, portfolio and risk-free returns (twelve months). The use of different time periods for the calculation of beta, returns and Equity Risk Premium (calculated over a minimum period of thirty-five years) induce an inconsistency in the estimates and of course leads to numbers with no meaning. The author's methodology is outlined as follows:

1. Calculate the excess return of each decile portfolio as the difference between the twelve-month period return and the cumulative risk-free return over the same twelve-month period.
2. Estimate the beta for each decile using monthly returns for three years.
3. Calculate the historical equity risk premium (ERP) for the period from 1926 to the current year. The earliest estimate of the *PCSP* should be for 1961 to guarantee at least thirty-five years of data of the ERP.
4. Calculate the expected return as beta multiplied by ERP.
5. *PCSP* is the difference between the twelve-month realized return calculated in step 1 and the expected return calculated in step 4.

In short, the *PCSP* use the same equation as the *SP*: $SP = r_p - (r_f + \beta \times ERP)$ where r_p and r_f are calculated over the last twelve-month period and β over the three-year look-back period and *ERP* is the historical ERP since 1926. These inconsistencies between the dates used for the calculation of *PCSP* render it useless because it does not really capture the actual *SP* but a deviation of the return from a long-term expectation using a short term estimated beta.

Estimation issues in PCSP

Valuation professionals use cost of capital to discount projects with long term life spans. The only logical thing is to use long term estimates of the cost of equity and for the *SP* as well. The use of three years and twelve months to calculate the alternative *SP* is inconsistent.

The author argues that using a three-year period for estimating beta of different portfolios based on size (i.e., deciles) is more appropriate because it reflect the up to date information. However, the choice of the estimation period depends on the objective of the estimation. For example, when we are studying the impact of an event (event study) we use daily returns and a typical look-back period for estimating the firm's beta of 250 days.

In the case of *SP*, we are estimating a "risk premium"[31] to be used to discount cash flows of projects with a long-term life spans. We need to estimate a stable and long-term market risk premium, which implies the need for a long enough period of estimation. The author is contradicting himself when he is using an ERP of a minimum thirty-five years and expanding his window and assumes that the *SP* should be calculated using twelve-month returns and a relatively short estimation window.[32] The three-year look-back period is too noisy for estimating beta, and it picks up short-term variation in the portfolio returns.

To illustrate, we estimated betas for both large and small firm deciles using returns from Professor Kenneth French's website.[33] We calculated monthly beta following the methodology proposed by the author (three-year look-back period) and monthly betas using an expanding window as it is done in Duff & Phelps *Valuation Handbook*. The results are plotted in Figure 1. The moving three-year estimation period proposed by the author is noisy and shows that betas of small cap stocks fluctuate between a low of 0.58 and a high of 1.7; where betas estimated using an expanding window from 1926 to current has a low of 1.42 and a maximum of 1.64. This means that a portfolio of small firms with a beta of 0.58 has expected return of 48% lower than the market portfolio. Does this mean that a portfolio of small cap stocks is a defensive portfolio?

Imagine that a Chief Financial Officer is using this methodology to calculate the net present value (NPV) of a project with the same size as a firm in the lowest decile. The obtained required rate of return would be nearly half what would be expected from the market portfolio and possibly resulting in accepting projects that should be rejected. The estimates of the three-year look-back beta are affected by market conditions which makes estimating the long term expected return of CAPM impossible. These movements are temporary and would bias our estimates of a long-term cost of equity for a long-term project or simply for company with indefinite life. While

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[31] There is still an ongoing debate about what the *SP* is. According to Fama and French (Eugene F. Fama and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *The Journal of Finance* 47 (1992):427–465; "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics* 33 (1993):3–56; "Multifactor Explanations of Asset Pricing Anomalies," *The Journal of Finance* 51 (1996):55–84), it is a risk premium. According to Daniel and Titman, Kent Daniel, Mark Grinblatt, Sheridan Titman, and Russ Wermers ("Measuring Mutual Fund Performance with Characteristic-Based Benchmarks," *The Journal of Finance* 52 (1997):1035–1058), it is not a compensation for risk. This discussion is beyond the scope of this paper.

[32] The author cites Yahoo finance website as a reference for the use of a three-year look-back period. Most finance textbooks recommend the use of five years as a rule of thumb for the look-back period.

[33] Accessed at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, August 8, 2018.

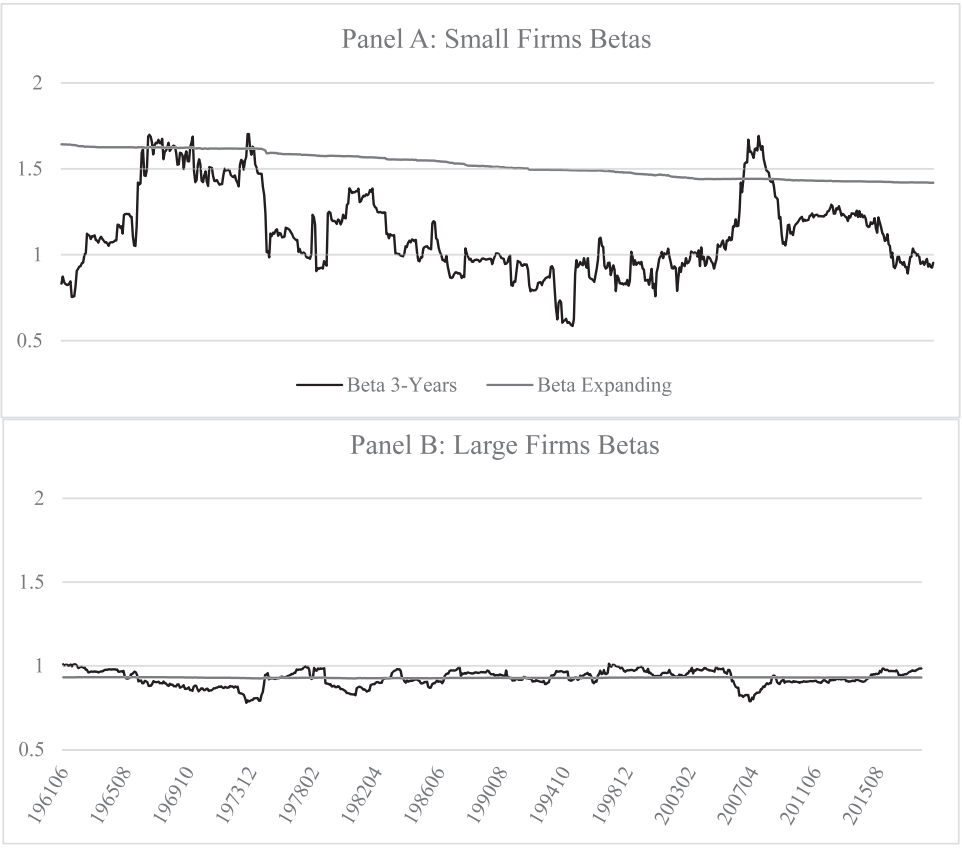

Figure 1

Low and Large Size Portfolios' BetasEstimates of Beta over the Period June 1961 to June 2018 Using a Three-Year Moving Look-Back Period and an Expanding Look-Back Period Starting June 1926 for Low and Large Size Portfolios

using a short look-back period may be useful for day traders, valuation professionals are typically estimating underlying value based on long-term prospects. The methodology the author is proposing is picking up the statistical errors that he was set to avoid by proposing the same methodology.

The volatility of the *PCSP* estimate increases even further with the proposed use of twelve-month period in calculating return of small cap and large cap portfolios and the return of a riskless asset. Choosing a twelve-month period does not represent a good proxy for long term returns of the portfolios. The objective of the author is to have returns that reflect the current market conditions, but these are not market conditions that will prevail in the future and one should be careful when engaging in this exercise.

Other Criticisms

Risk Premium Report results embody hindsight

Some critics have suggested that the returns reported in the Risk Premium Report are the result of embodying hindsight. All categorization of companies for the Risk Premium Report dataset and portfolios is based on data known before the beginning of each year for which returns are calculated.

In determining companies to exclude from the base dataset (e.g., companies lacking five years of publicly traded price history or companies with a negative five-year average EBITDA for the previous five years) all data for the selection process are known before the beginning of any year. The exclusion of companies based on historical financial performance does not imply any unusual foresight on the part of hypothetical investors in these portfolios. In forming portfolios to calculate returns for a given year, we exclude companies on the basis of performance during previous years (e.g., average net income for the five prior fiscal years), rather than current or future years. Portfolio ranking based on size characteristics in the Risk Premium Report—Size Study are always determined by size measures preceding the annual period in which the observed returns are measured.

For returns measured in 2009, for example, companies ranked by size measured by five-year average net income

are placed in portfolios based on their five-year average net income for the period of 2004–2008. This procedure means that there is no hindsight built into the formation of the size ranked portfolios.

Thus, investors choosing to apply the methodology employed in the Risk Premium Report could select companies to include in their portfolios using the same selection criteria as is used in building the portfolios as are reported in the Risk Premium Report each year.

Returns used in the Risk Premium Report–Size Study are not realistic

Some critics claim that the average returns reported in the Risk Premium Report–Size Study differ from those of some funds targeted at investing in small-cap companies and therefore are unrealistic estimates of expected returns.

In estimating the expected return from an investment in the stock market, one can summarize historic returns by calculating a geometric average return or an arithmetic average return. The arithmetic average of prior period returns is always greater than the geometric average of returns during the same period because it captures the volatility of the realized returns. Both are correct summaries of observed return data.

In cases where an analyst is estimating the expected accumulated wealth at a point in the future, the preferred estimate of the rate of return on an investment in the stock market is the geometric mean of prior period returns or the implied forward return. But if the analyst is estimating the discount rate that should be used in discounting expected cash flows in future years, the preferred statistic is the arithmetic average of realized returns (realized risk premiums) or the arithmetic average equivalent of implied future returns.[34]

The Risk Premium Report reports the arithmetic average of prior period returns because the data is intended to be used in developing discount rates to be used in discounting expected cash flows in future years.

One commentator compared the arithmetic average returns reported in the Risk Premium Report for 1963 through 2012 to the geometric average returns reported for a fund (DFSCX), which invests in small-cap stocks for the period 1981 through 2015. Any analyst will recognize that this this is a flawed comparison.

The more proper comparison would be to compare the arithmetic average return on the DFA microcap fund for the period 1963 through 2012 with the returns reported in the Risk Premium Report for the same period.

First, to obtain the returns for the period 1963 through 2012, we analyzed the Ibbotson Associates SBBI US Small Stock TR USD index, which is constructed as follows[35]:

- *DFA U.S. Micro Cap Portfolio* (April 2001 to December 2016), the small-cap stock return series is the total return achieved by the net of fees and expenses. At year-end 2016, the DFA U.S. Micro Cap Portfolio contained 1,546 stocks, with a weighted average market cap of $1.128 billion.
- *DFA U.S. 9–10 Small Company Portfolio* (January 1982–March 2001; renamed the DFA U.S. Micro Cap Portfolio in April 2001). The fund's target buy range was a market-cap-weighted universe of the ninth and 10th deciles of the NYSE, plus stocks listed on the NYSE Amex (now the NYSE MKT) and NASDAQ National Market.
- The equities of smaller companies from 1926 to 1980 are represented by the historical series developed by Banz.

We can more properly compare the returns for the Ibbotson Associates SBBI US Small Stock TR USD index (essentially the DFA fund used in the commentator's critique) to the returns reported in the Risk Premium Report.

Second, we can compare the size of the companies included in each. The average market capitalization reported in Appendix A-1 for portfolio 25 in the *2013 Risk Premium Report* is $94 million.

At year-end 2012 (the data-through date of the *2013 Risk Premium Report* is December 31, 2012), the DFA U.S. Micro Cap Portfolio contained 1,957 stocks, with a weighted average market capitalization of $682 million[36] (the data-through date of the *2013 SBBI* "*Classic*" *Yearbook* is also December 31, 2012).

The size of companies reported for the DFA U.S. Micro Cap Portfolio should therefore be more properly compared with portfolio 21 where the average market capitalization reported in Appendix A-1 for portfolio 21 in the *2013 Risk Premium Report* is $811 million.

The average annual arithmetic return reported in Appendix A-1 for portfolio 21 in the *2013 Risk Premium Report* is 16.78%.

The average market capitalization reported Appendix A-1 for portfolio 22 in the *2013 Risk Premium Report* is $615 million. The average annual arithmetic return reported in Appendix A-1 for portfolio 22 in the *2013 Risk Premium Report* is 17.21%.

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[34] "Using the Geometric Average for Compounding and the Arithmetic Average for Discounting," in *Cost of Capital: Applications and Examples* 5th ed. (City: Publisher, year), 153–159.

[35] *2017 SBBI Yearbook*, Chapter 3, "Description of the Basic Series," pages 3.2 and 3.3.

[36] *2013 SBBI* "*Classic*" *Yearbook*, Chapter 3, "Description of the Basic Series," page 54.

The average annual returns reported for the Risk Premium Report portfolios (21 and 22; 16.78% and 17.21%, respectively) that have similar average market capitalizations ($811 million and $615 million, respectively) as the DFA average market capitalization as of December 31, 2012 ($682 million) are similar (the DFA series' arithmetic average annual return 1963–2012 is 16.2%).

The small differences in returns (16.2% versus 16.78% and 17.21%) may be attributed to many factors (e.g., the Risk Premium Report portfolio returns are equally weighted, so the smaller companies' intra-portfolios are given equal weight with the larger companies' intra-portfolios; this weighting will tend to make the Risk Premium Report portfolios have greater returns, all other things held the same, than the market weighted returns reported for the DFA fund).

Small company stocks do not always outperform large company stocks

Small companies are believed to typically have greater expected rates of return compared to large companies because small companies are inherently riskier. However, this leaves the question of why small-stock returns have not *consistently* outperformed large-company stocks for various periods. We observe that the size effect is *cyclical*. Readers of the *SBBI Yearbooks* have long been aware that the small-stock premium (returns of small-cap companies versus large-cap companies) tends to move in cycles, with periods of negative premia followed by periods of high premia. It has been suggested that periods in which small-cap firms have outperformed large-cap firms have generally coincided with periods of economic growth. At least one study contends that the variability in the size effect over time is predictable because large-cap firms generally outperform small-cap firms in adverse economic conditions. Credit conditions are exceedingly important for all firms, but especially for small firms. Small firms generally are at a disadvantage when it comes to financing, and suppliers of debt capital are less likely to lend to small firms in periods of adverse economic conditions.[37] Further, since the late 1990s, many companies have faced a perceived lack of pricing power. In this type of environment, small firms are likely to be at a disadvantage.[38]

For these reasons, analysts should not be surprised to find small-cap stocks underperforming large-cap stocks for lengthy periods of time. The cyclicality is part of the risk of small companies; if small companies *always* earned more than large companies, small companies *would not* be riskier in the aggregate. In a recent study, F-F find that the estimated probabilities that small-cap companies can be expected to underperform large-cap companies over a five-year investment horizon is only 29.8%.[39] But they also find that as the investment horizon increases, the likelihood that the returns on small-cap companies will exceed returns of large-cap companies increases:

> In short, value and small stock premiums over Market are always risky, but for longer return horizons, good outcomes become more likely and more extreme than bad outcomes.

One can argue that advocates of the size effect can find satisfaction in the erratic performance of small-cap stocks. If you believe that small-company stocks are riskier than large-company stocks, then it probably follows that small-company stocks should not always outperform large-company stocks in *all* periods. This is true even though the expected returns are greater for small-cap stocks over the long-term.

By analogy, bond returns occasionally outperform stock returns. For example, in 2007, 2008, 2011, and 2014, long-term U.S. government bonds significantly outperformed large-cap company stocks (total return on bonds equaled 9.9% compared to the return on large cap stocks of 5.5% in 2007; 25.9% compared to –37.0% in 2008; 27.1% compared to 2.1% in 2011; and 24.7% compared to 13.7% in 2014), yet few would contend that over longer horizons the expected return on bonds is greater than the expected return on stocks (for the entire period 2007 through 2016, the total returns on long-term U.S. government bonds was less than the returns on large-cap stocks, 6.5% compared to 6.9%).[40]

Richard Bernstein, a well-known market observer wrote:

> An important question that is not answered by the doubters of the small stock effect is why smaller capitalization stocks have had performance cycles at all.[41]

Size Effect is inconsistent with the theory of CAPM

Some argue that the size effect lacks a theoretical basis. First, researchers of the size effect never have claimed

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[37] Ching-Chih Lu, "The Size Premium in the Long Run," Working paper, December 2009. The author reports on a study he conducted comparing the average market values of common equity between companies with investment-grade credit ratings and those with non-investment-grade credit ratings for the period 1994–2008. He found that the companies with better credit ratings were nine to ten times larger than the companies with poorer credit ratings.

[38] Satya Dev Pradhuman, *Small-Cap Dynamics: Insights, Analysis, and Models* (New York: Bloomberg Press, 2000), 23–28.

[39] Eugene F. Fama and Kenneth R. French, "Volatility Lessons," Working paper (November 2017).

[40] *2017 SBBI Yearbook*.

[41] Richard Bernstein, *Style Investing: Unique Insights into Equity Management* (New York: John Wiley & Sons, 1995), 142.

that the size effect is anything more than empirical evidence that the CAPM beta does not capture all systematic risk. Critics of the size effect by implication hold that the CAPM beta is the only systematic risk premium. But research has shown that realized returns are not consistent with the returns predicted by the textbook CAPM and its sole risk measure, beta.

F-F published two studies critical of beta. In one study they stated:

> The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market betas (the slope in the regression of a security's return on the market's return), and (b) market betas suffice to describe the cross-section of expected returns.

F-F observed that the relationship between market beta and average return is flat.[42] In a follow-on study, they found that problems with CAPM using U.S. data show up in the same way in the stock returns of non-U.S. major markets.[43]

As authors of one book put it:

> Fama and French significantly damaged the credibility of the CAPM and beta.[44]

CAPM stipulates that expected return on an asset is linearly related *only* to its beta. The model underpins the status of academic finance, as well as the belief that asset pricing is an appropriate subject for economic study. But CAPM has failed the test of reality. Dempsey reexamined the research of Black et al., which did much to lay the empirical foundation for the CAPM. He found that the data do not actually provide a justification of the CAPM as claimed.[45]

Dempsey also criticizes the CAPM foundation that markets are fundamentally rational; empirical evidence casts doubt on the Efficient Market Hypothesis (EMH) and Rational Expectations Hypothesis (REH).

EMH holds that financial prices reflect all available information relevant to the values of the underlying assets—price of an asset converges on its value fairly quickly. The finance industry interpreted EMH to imply market is capable of pricing financial assets correctly and that deviations from fundamental values could not persist.

However, the disappearance of buyers during the Financial Crisis was a severe blow to EMH and Professor Fama, who introduced the EMH, which indirectly criticizes EMH by declaring CAPM is dead.

The REH precludes heterogeneity of expectations is disputed in the literature. The author concludes that CAPM, EMH, and REH are all about "incredible" assumptions.

When researchers test the CAPM cost of equity estimates, they find that realized returns for high-beta stocks are too high (relative to returns predicted by CAPM), and they find that realized returns for low-beta stocks are too low (relative to returns predicted by CAPM). The implications of this work are that if CAPM betas do not suffice to explain expected returns, the market portfolio is not efficient. If this implication is true, then CAPM has potentially fatal problems.

> we argue that the CAPM fails as a paradigm for asset pricing. . . . a reexamination of the research of Black et al.,[46] which did much to lay the empirical foundation for the CAPM, reveals that the data do not actually provide a justification of the CAPM as claimed, but rather constitute confirmation of the null hypothesis, namely that investors impose a single expectation of return on assets. Researchers, however, did not wish to abandon the core paradigm of market rationality. Such paradigm, after all, justified the status of finance as a subject worthy of "scientific inquiry."[47]

More problematic, researchers have shown that stock returns are not normally distributed—a finding that in and of itself demonstrates that beta cannot be the sole measure of risk.[48] The studies have found that distributions of stock returns are skewed[49] and have fatter tails than a normal distribution. Many critics of CAPM hold that the finding of non-normalcy of returns alone invalidates CAPM. A reasonable likelihood function explaining the distribution of returns as time goes to infinity is the Cauchy distribution; it is a better model of reality than

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[42] Eugene Fama and Kenneth French, "The Cross-Section of Expected Stock Returns," Working paper.

[43] Eugene Fama and Kenneth French, "Value versus Growth: The International Evidence," *Journal of Finance* (December 1998): 427–465.

[44] Tim Marc Goedhart Koller, and David Wessels, *Valuation—Measuring and Managing the Value of Companies*, 5th ed. (Hoboken, NJ: John Wiley & Sons, 2010): 256.

[45] Mike Dempsey, "The Capital Asset Pricing Model (CAPM): The History of a Failed Revolutionary Idea in Finance?" *Abacus* 49, Supplement (2013):7.

[46] Michael C. Jensen, Fischer Black, and Myron Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in M. Jensen (ed.), *Studies in the Theory of Capital Markets* (New York: Praeger Publishers, 1972), xx–xx.

[47] Mike Dempsey, "The Capital Asset Pricing Model (CAPM): The History of a Failed Revolutionary Idea in Finance?" *Abacus* 49, Supplement (2013):8.

[48] Hsing Fang and Tsong-Yue Lai in "Co-Kurtosis and Capital Asset Pricing," *Financial Review* (May 1997):293–307, derive a four-moment CAPM and show that systematic variance, systematic skewness, and systematic kurtosis contribute to the risk premium, not just beta; Fred Arditti in "Risk and the Required Return on Equity," *Journal of Finance* (March 1967):19–36, demonstrates that skewness and kurtosis cannot be diversified away by increasing the size of the portfolios.

[49] Skewness describes asymmetry from the normal distribution in a set of statistical data. Skewness can come in the form of negative skewness or positive skewness, depending on whether data points are skewed to the left (negative skew) or to the right (positive skew) of the data average.

either the normal or the log-normal distribution. But among the characteristics of the Cauchy distribution is that it has no mean; consequently, the central limit theorem does not hold which negates mean-variance finance as we know it (i.e., it negates CAPM, the Arbitrage Pricing Theory [APT], and the Black-Scholes Option Pricing Model).

F-F believe that the results of their papers point to the need for pricing risk using a model that is not dependent on beta alone because beta as traditionally measured is not a complete description of an asset's risk.[50] F-F introduced first a three-factor model and, later, a five-factor model intended to explain stock prices. Both models are empirical in nature and not built upon an underlying theory, and clearly not built upon the textbook CAPM.

The state of our understanding of how asset risk is priced is summed up by Professor John Cochrane:

> Discount rates vary a lot more than we thought. The puzzles and anomalies that we face amount to discount rate variation we don't understand. Our theoretical controversies are about how discount rates are formed… Theories are in their infancy.[51]

Those practitioners that include size premia in their estimates of the cost of equity when applying the MCAPM are simply applying an empirically observed correction to the defective textbook CAPM. Why should practitioners be faulted for correcting a defective model?

There continue to be critiques of the size effect in the academic literature most often focused on the difficulty of using the size effect in building portfolios and effectively implementing a profitable trading strategy, not focused on estimating the cost of capital.[52]

Considerations in Applying a Size Premium

As displayed in the accompanying exhibits, the size effect has been observed even when looking at recent periods starting in 1981 and 1990. If one holds that you should not apply the *SP* in the MCAPM and that beta should be the only measure of risk, one is supporting using the pure or textbook CAPM to estimate expected returns. But that cannot be correct as the literature clearly demonstrates. Though the pure CAPM is a good tool to teach the relationship of risk and return, pure CAPM is not an effective model for estimating expected returns.[53] Despite the empirical evidence, there are some who blindly support the pure CAPM. This author disagrees and concludes that until we have better models for pricing risk, one should consider using the MCAPM instead of the pure CAPM in developing discount rates.[54] Applying the *SP* is based on observed returns, not a theoretical ideal. Therefore, one can match the subject company's characteristics to the companies that had similar characteristics over time and then use the observed returns for the latter as proxy to what expected returns might be for the subject company.

However, when applying the *SP* in estimating the cost of equity capital for a small company, one should not simply apply it by rote. One should be matching the characteristics of the subject company with those of the companies used in arriving at, say, the *SP*.

For example, some practitioners gravitate to using the *SP* observed for subdecile 10z without comparing the characteristics of the subject company for which they are estimating the cost of capital with those of the companies that comprise subdecile 10z.

Examining the fundamental characteristics of the companies comprising subdecile10z (see Exhibit 9), one finds that subdecile 10z is partially populated with troubled companies.[55]

One way to correct for the underestimation of beta is by using sum beta method[56] instead of the OLS method. Stocks of smaller companies generally trade less frequently and exhibit more of a lagged price reaction (relative to the market benchmark index) than do large cap stocks. The sum beta estimates are generally *greater* for smaller companies than the betas derived using non-lagged market benchmark data, therefore resulting in size premia of *smaller* magnitude. Though using the sum beta estimate increases the beta for smaller market cap companies, we still observe premia in excess of that predicted by beta for smaller companies. For example, for the period ending December 31, 2016, the beta estimates and *SP* for the CRSP Micro-Cap size grouping (deciles 9 and 10 combined) were 1.35 and 3.67% using OLS

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[50] Eugene Fama, and Kenneth French, "The Cross-Section of Expected Stock Returns," Working paper.

[51] John C. Cochrane, University of Chicago Booth School of Business, "Discount Rates," American Finance Association Presidential Address (January 8, 2011), accessed at http://faculty.chicagobooth.edu.john.cochrane/research/papers.

[52] For example, see Ron Alquist, Ronen Israel, and Tobias Moskowitz, "Fact, Fiction, and the Size Effect," *Journal of Portfolio Management* (forthcoming). Many of these criticisms are addressed in Chapter 15, "Criticisms of the Size Effect," *Cost of Capital: Applications and Examples* 5th ed.

[53] Chapter 13, "Criticism of CAPM and Beta versus Other Risk Measures," *Cost of Capital: Applications and Examples* 5th ed.; Pablo Fernandez, "CAPM: An Absurd Model," *Business Valuation Review*, 34(1) (Spring 2015):4–23, accessed at http://ssrn.com/abstract=2505597, April 13, 2015; M. Dempsey, "The Capital Asset Pricing Model (CAPM): The History of a Failed Revolutionary Idea in Finance?" *Abacus* 49, Supplement (2013):8.

[54] At least when conducting valuation analyses denominated in U.S. dollars from the perspective of a U.S. dollar investor.

[55] Originally published as Exhibit 4.10 in the 2017 Duff & Phelps *Valuation Handbook – U.S. Guide to Cost of Capital*.

[56] See Chapter 11, "Beta: Differing Definitions and Estimates," *Cost of Capital: Applications and Examples* 5th ed.

Exhibit 9

Breakdown of Subdecile 10z Companies: Market Value of Equity between \$2.516 and \$73.504 Million (Appeared as Exhibit 4.10 in the Duff & Phelps *2017 Valuation Handbook – U.S. Guide to Cost of Capital*) as of September 30, 2016

Subdecile 10z	Market Value of Equity (in \$Millions)	Book Value of Equity (in \$Millions)	Five-Year Average Net Income (in \$Millions)	Market Value of Invested Capital (in \$Millions)	Total Assets (in \$Millions)	Five-Year Average EBITDA (in \$Millions)	Sales (in \$Millions)	Return on Book Equity (%)	OLS Beta	Sum Beta
95th Percentile	70.114	85.864	4.600	176.784	660.828	22.767	248.603	15.5	2.71	3.45
75th Percentile	53.104	42.925	0.709	72.135	106.517	3.179	67.029	2.5	1.35	1.79
50th Percentile	34.343	18.007	(3.958)	46.750	40.658	(1.547)	25.303	(23.0)	0.70	1.01
25th Percentile	18.846	5.866	(13.929)	25.493	17.456	(9.474)	8.086	(94.5)	0.28	0.43
5th Percentile	6.657	(5.338)	(25.146)	9.759	6.319	(18.669)	1.329	(222.4)	0.09	0.13

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2017 Center for Research in Security Prices (CRSP) University of Chicago Booth School of Business, (ii) S&P Research Insight. Used with permission. All rights reserved Calculations performed by Duff & Phelps, LLC.
EBITDA = earnings before interest, taxes, depreciation, and amortization.

regression of look-back data compared to 1.59 and 2.01% using the sum beta estimate.[57]

Even using sum beta over a look-back period may underestimate a forward-looking beta for a troubled company. The market price of the troubled company stock likely is readjusting downward to the troubled nature of the subject company even during periods when the general stock market returns are increasing.[58]

For example, when examining companies comprising the subdecile 10z as of September 30, 2016, ranking them from largest to smallest in terms of their average net income for the prior five years, we find that the mid-point of the ranked companies (i.e., the 50th percentile) has an average net income equal to a loss of \$3.958 (millions). While the overall sum beta of the subdecile 10z equals 1.64 compared to the OLS estimate of 1.28, both of those estimates (and the resulting size premia) suffer from the inclusion of troubled companies. The beta estimates of the companies comprising the 10z subdecile are less than the beta estimates of the subdecile 10a (the largest 50% of companies comprising decile 10), which is comprised of fewer troubled companies.

From these data we can conclude the following:

- Using the OLS method of estimating betas for calculating the *SP* for subdecile 10z generally *understates* betas and, therefore, may *overstate* the *SP*.
- Subdecile10z is populated by many *large* as measured by total assets (but highly leveraged) companies with *small* market capitalizations that probably do not match the characteristics of financially healthy but small companies (as indicated by the percentage of equity to total assets of the 95th percentile of companies, the largest companies, comprising the subdecile).
- Stocks of *troubled* companies included in the data may have had their stock prices so diminished that they are likely trading like call options (unlimited upside, limited downside) (as indicated by the negative latest fiscal year return on book equity of the 25th percentile and fifth percentile of companies comprising the subdecile). Even if one uses the sum beta method, the beta estimates are likely underestimated and the SP overstated.

Let us contrast the composition of companies in subdecile 10z with those that comprise the 25th portfolio (comprising the smallest companies) of the Risk Premium Report-Size Study.

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57 Exhibit 4.7, 2017 Duff & Phelps *Valuation Handbook – U.S. Guide to Cost of Capital*.

58 "Cost of Capital Equity Considerations," in Chapter 17, "Distressed Businesses," *Cost of Capital: Applications and Examples* 5th ed.

Exhibit 10

Size Measure of Companies That Comprise Portfolio 25 of the *Risk Premia Report* December 31, 2016 (appeared as Exhibit 4.11 in the Duff & Phelps *2017 Valuation Handbook – U.S. Guide to Cost of Capital*)

Portfolio 25	Market Value of Equity (in $millions)	Book Value of Equity (in $Millions)	Five-Year Average Net Income (in $Millions)	Market Value of Invested Capital (in $Millions)	Total Assets (in $Millions)	Five-Year Average EBITDA (in $Millions)	Sales (in $Millions)	Number of Employees
Largest Company	$291.526	$159.204	$11.274	$387.281	$346.465	$39.386	$307.857	686
95th Percentile	260.490	151.276	10.302	355.585	318.170	36.761	276.458	641
75th Percentile	189.941	116.879	7.292	257.179	249.862	26.608	200.273	485
50th Percentile	99.386	74.349	4.148	141.789	163.579	15.120	118.960	272
25th Percentile	48.289	33.630	1.976	67.336	62.163	6.991	55.810	131
5th Percentile	16.848	13.868	0.669	20.342	24.577	2.109	18.314	10
Smallest Company	4.784	4.756	0.054	8.190	9.283	1.130	2.619	1

Sources of underlying data: (i) CRSP U.S. Stock Database and CRSP U.S. Indices Database © 2017 Center for Research in Security Prices (CRSP) University of Chicago Booth School of Business, (ii) S&P Research Insight. Used with permission. All rights reserved Calculations performed by Duff & Phelps, LLC.

EBITDA = earnings before interest, taxes, depreciation, and amortization.

The Risk Premium Report studies use the sum beta method (using monthly returns over a look-back period) to measure the *SP* because we observe that the betas of small companies in the data set are underestimated if one uses the OLS method of estimating betas because of the low liquidity of small company stocks.[59]

The characteristics of the companies comprising the portfolio of the smallest companies (Portfolio 25) can be thought of as profitable (not troubled), but simply small (as shown in Exhibit 10).

The information and data in the Risk Premium Report studies are primarily designed to be used to develop cost of equity capital estimates for the large majority of companies that are fundamentally healthy, and for which a "going concern" assumption is appropriate.

One can further refine the appropriate *SP* by comparing risk characteristics of the subject firm with those of the companies comprising the portfolio of companies reported in the Size Study. Exhibits 5, 6, 7, and 8 display the fundamental risk characteristics that match the relative portfolio based on size. Note that the differences in debt do not cause the differences in risk. Also note that the unlevered betas of the portfolios comprising smaller companies are greater than those of larger companies; the average operating margins of the companies comprising the portfolios of smaller companies are less than the average operating margins of those of the larger companies; and the coefficient of variation of operating margins of the companies comprising the portfolios of smaller companies are greater than the coefficient of variation of operating margins of those of the larger companies. These fundamental risk measures support the position that smaller companies are on the average riskier than larger companies. The *SP* may be a proxy for fundamental risk differences.

Some subject companies have fundamental risk characteristics that point the analyst to use either a lesser or a greater *SP* than the published portfolio *SP* measures might at first indicate. For example, the variation of operating margin may be less for the subject company than for the typical company of equal size as measured by say total assets. In that case, the analyst should most likely apply a lesser *SP* for the subject company.[60] The goal is to match the *SP* that is appropriate given the fundamental risk characteristics of the subject company.

As discussed previously, the relative returns on small company stocks compared to returns on large company stocks are cyclical. This leads one to ask if one should

[59] Ang erroneously criticized the Risk Premium Report for using annual returns in estimating beta despite the fact that the methodology is clearly explained.

[60] Data on average operating margin for each size-based portfolio is available through the Duff & Phelps Cost of Capital Navigator platform.

apply a *SP* if the stock markets indicate that the multiples for small cap stocks have declined compared to large cap stocks? In other words, in a situation where the returns for small cap companies in the immediately preceding period were less than the returns for large cap stocks should one not use a *SP* in estimating the cost of equity capital for a small company? We believe that the valuation professional should include *SP* even in these times because the cost of equity capital should be based on the *expected returns*. In developing a cost of equity capital estimate when valuing a non-publicly traded business, the valuation professional is not mimicking a trader. The trader is dealing with small blocks of stock and high liquidity. We are estimating an expected return over a long holding period.

Conclusion

Academic and empirical evidence indicate that the pure textbook CAPM is an imperfect indicator of expected returns. Until better models become more accepted and easier for the valuation professional to use,[61] the MCAPM will likely continue to be widely used by valuation professionals. *SP* help the valuation professional correct the pure CAPM for the risks of smaller companies not captured by beta. In this paper, I demonstrate that the methodology followed by Duff & Phelps to calculate *SP* is robust and yields a consistent stable premium to be used for pricing long term project as it should be for a good measure of cost of capital.

[61] For example, see Eugene F. Fama and Kenneth R. French, "A Five-Factor Asset Pricing Model," *Journal of Financial Economics* 116 (2015):1–22.