DUFF&PHELPS

Risk Premium Report 2013

Selected Pages and Examples
(Data Exhibits not included)

This document is an excerpt of the *2013 Risk Premium Report,* and includes an overview of the methodologies employed in performing the analysis required for the *Size Study*, *Risk Study*, *High-Financial-Risk Study*, and proper use of the *"C" Exhibits* that constitute the *Risk Premium Report*. The excerpt also includes a limited number of examples demonstrating how the *Risk Premium Report*'s size premia and risk premia data can be used to estimate cost of equity capital (more examples are available in the full version *Report*). The excerpt does not include the size and risk premia data exhibits that are available in the full version of the *Risk Premium Report*.

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To download a free copy of *"Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of 'Flight to Quality'"* by Roger J. Grabowski, visit:

www.DuffandPhelps.com/CostofCapital

To learn more about the latest theory and practice in cost of capital estimation, including cost of capital for uses in business valuation, project assessment and capital budgeting, divisional cost of capital, reporting unit valuation and goodwill impairment testing, valuing intangible assets for financial reporting, and transfer pricing, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010).

"This book is the most incisive and exhaustive treatment of this critical subject to date." – Stephen P. Lamb, Esquire; Former Vice Chancellor, Delaware Court of Chancery

Additional copies of the *2013 Duff & Phelps Risk Premium Report* may be obtained from our Distributors:

Business Valuation Resources (BVR)
www.bvresources.com/DP
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The *Duff & Phelps Risk Premium Report* is intended to be used as a companion publication to the web-based *Duff & Phelps Risk Premium Calculator*. The *Duff & Phelps Risk Premium Calculator* is available through Business Valuation Resources (BVR) and ValuSource.

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NOTE: The table of contents shown here is the table of contents of the full *2013 Risk Premium Report*, and is included so that readers can view the contents of the full *Report*. It is not the table of contents for this document. Footnotes and page references are in the context of the full *Report*.

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Introduction

Who Should Use the Duff & Phelps Risk Premium Report

The *Duff & Phelps Risk Premium Report* ("*Risk Premium Report*", or "*Report*") is designed to assist financial professionals in estimating the cost of equity capital ("COE") for a subject company. Cost of equity capital is the return necessary to attract funds to an equity investment. The risk premia and size premia calculated in the *Report* can be used to develop COE estimates using both the buildup method and the Capital Asset Pricing Model (CAPM).

In addition to the traditional professional valuation practitioner, the *Risk Premium Report*, and the accompanying online *Duff & Phelps Risk Premium Calculator* ("*Risk Premium Calculator*", or "*Calculator*"), are designed to serve the needs of:

- Corporate finance officers for pricing or evaluating mergers and acquisitions, raising private or public equity, property taxation, and stakeholder disputes.

- Corporate officers for the evaluation of investments for capital budgeting decisions.

- Investment bankers for pricing public offerings, mergers and acquisitions, and private equity financing.

- CPAs who deal with either valuation for financial reporting or client valuations issues.

- Judges and attorneys who deal with valuation issues in mergers and acquisitions, shareholder and partner disputes, damage cases, solvency cases, bankruptcy reorganizations, property taxes, rate setting, transfer pricing, and financial reporting.

Appropriate Use of the Duff & Phelps Risk Premium Report

The information and data in the *Risk Premium Report* (and in the online *Risk Premium Calculator*) is primarily designed to be used to develop cost of equity capital estimates for the large majority of companies that are fundamentally healthy, and for which a "going concern" assumption is appropriate. "High-financial-risk" (i.e. "distressed") companies are excluded from the base dataset and analyzed separately.

Because financial services companies are excluded from the base set of companies used to develop the analyses presented in the *Report*, the *Report* (and the online *Risk Premium Calculator*) should not be used to estimate cost of equity for financial services companies. Financial services companies include those companies in finance, insurance, or real estate (i.e. companies with an SIC Code that begins with "6").

While the data used in the *Risk Premium Report* (and in the on-line *Risk Premium Calculator*) are drawn from information on public companies and, therefore, the resulting COE estimates using the data are "as if public", the *Report* and *Calculator* can be used to develop estimates of COE for divisions, reporting units and closely held businesses without "guessing" at the value of the business before one begins the analysis. Rather, fundamental measures of firm size (e.g., sales, net income, EBITDA) and risk (e.g., operating margin) can be used to directly estimate COE for non-public businesses.

Introduction

History of the Duff & Phelps Risk Premium Report

In 1990, Roger Grabowski began closely studying the relationship between company size and stock returns.[1] Grabowski's early research focused on size as measured by market capitalization, but quickly advanced to two additional areas of inquiry: whether stock returns were predicted by measures of size other than market capitalization, and whether stock returns were predicted by fundamental risk measures based on accounting data. To investigate these questions, in 1992 Grabowski, working with a colleague[2], contracted with the Center for Research in Security Prices (CRSP) at the University of Chicago to build a database that combined stock prices, number of shares, and dividend data from the CRSP database with accounting and other data from the Standard & Poor's *Compustat* database.

What they found was that as size decreases, or risk increases (as measured by fundamental accounting data), returns tend to increase (and vice versa). Thereafter, they published a series of articles reporting their findings, culminating with a seminal 1996 article and a subsequent article in 1999 which together serve as the foundation of the *Duff & Phelps Risk Premium Report*.[3]

The *2013 Duff & Phelps Risk Premium Report* includes data available through December 31, 2012, and should be used for calendar year 2013 valuations.

Recent Changes and Additions

Now in its 18th year of publication, the *Risk Premium Report* continues to be at the forefront in providing comprehensive valuation methodology and data.

The *2013 Risk Premium Report* includes a new, expanded section about using the *Report* to value small companies. This new section provides a listing of the smallest and largest companies in "Portfolio 25" (Portfolio 25 is comprised of the smallest companies) for each of the eight size measures (see Table 2 on page 22). This was added to give the valuation professional greater capability to gauge the size characteristics of his or her subject company relative to the size characteristics of companies that comprise Portfolio 25, and also provides support for adjustments to premia made by utilizing the "regression equation method" to estimate custom "interpolated" risk premia or size premia.

The development of the online *Duff & Phelps Risk Premium Calculator* in 2011 remains an important milestone in the history of the *Report*. The online *Risk Premium Calculator* makes using the *Risk Premium Report* even easier. The *Calculator* instantly delivers a fully customizable "Executive Summary" in Microsoft Word format that includes sourcing, key inputs, and a concluded range of cost of equity capital estimates using both the buildup and CAPM methods. In addition, a detailed record of all inputs, outputs, and calculations is exported to a "Support and Detail" Microsoft Excel workbook.[4]

In the *2012 Report*, we added a section entitled "Adjusting Risk Premium Report Data to Changing Economic Conditions" which discussed the pitfalls that one may encounter in pricing risk during these uncertain economic times. Two key inputs in COE estimates were discussed: the equity risk premium (ERP) and the risk-free rate (R_f). In the *2013 Report*, this section is updated, renamed "Overview of Duff & Phelps U.S. ERP and Matching Risk Free Rate" and moved to the new Appendix C. Appendix C provides a *high-level* summary of the Duff & Phelps U.S. ERP and matching Risk-Free Rate recommendations as of December 31, 2012 and February 28, 2013.

For a *detailed* discussion of the Duff & Phelps Equity Risk Premium and Risk-Free Rate recommendations, please visit www.DuffandPhelps.com/CostofCapital and download a free copy of "*Client Alert: Duff & Phelps Decreases U.S. Equity Risk Premium Recommendation to 5.0%, Effective February 28, 2013*".

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[1] Roger Grabowski, FASA, is a managing director in the Duff & Phelps Chicago office and part of the firm's Valuation Advisory Service practice. He is also co-author with Dr. Shannon Pratt of *Cost of Capital: Applications and Examples*, 4th Edition (John Wiley & Sons, 2010).

[2] David King, CFA, is National Technical Director of Valuation Services at Mesirow Financial Consulting, LLC. The research began when both he and Roger Grabowski were at Price Waterhouse, predecessor firm to PricewaterhouseCoopers.

[3] Roger J. Grabowski and David King, "New Evidence on Size Effects and Equity Returns", *Business Valuation Review* (September 1996, revised March 2000), & Roger J. Grabowski and David King, "New Evidence on Equity Returns and Company Risk", *Business Valuation Review* (September 1999, revised March 2000).

[4] The *Duff & Phelps Risk Premium Calculator* is available through Business Valuation Resources (BVR) and ValuSource.

Introduction

In 2012, we also added or expanded four other sections:

- **ERP Adjustment:** The ERP Adjustment is a necessary adjustment that represents the difference between the historical equity risk premium (ERP) used as a convention to calculate the various risk premia and size premia in the *Report*, and a user of the *Report's* own forward ERP estimate. The ERP Adjustment is *always* necessary when using "risk premia over the risk-free rate", but is *never* necessary when using "risk premia over CAPM" (i.e., size premia).

  NOTE: Why is properly applying the ERP Adjustment important? In cases where the ERP Adjustment is not applied (as indicated in Table 1 on page 14), the net effect is that the historical 1963–present ERP used in the calculations to create the *Report* is embedded in the COE estimate. By doing this, the *Report* user is in effect *adopting the historical ERP as measured over the 1963–present time period as his or her forward-looking ERP*, which may or may not be the ERP that the user wishes to use as of his or her valuation date. Please see "Proper Application of the Equity Risk Premium (ERP) Adjustment" on page 12.

- **The Size Effect:** An expanded examination of the size effect, and how the size effect changes over time. This discussion can be found on page 25.

- **Proper use of the "C" Exhibits:** An expanded discussion of a valuable capability of the *Risk Premium Report* – how to gauge whether an upward or downward adjustment to a risk premium or size premium (and thus, COE) is indicated, based upon the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived. This discussion can be found on page 112.

- **FAQ:** A frequently asked questions (FAQ) section that answers some of the most commonly asked questions about the *Report*. This new section can be found on page 133.

In the *2011 Report*, we improved the method of calculating unlevered premia, and added "smoothed" unlevered premia to Exhibits C-1 through C-8, and added unlevered premia to Exhibits D-1 through D-8 (unlevered premia are used to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt). We updated the unlevered premia published in the *2010 Report* using this improved method as well.[5]

Beginning with the *2011 Risk Premium Report*, the information previously reported in Exhibit E summarizing the size of the companies in Portfolio 25 for each of the eight alternative size measures was moved to the *Size Study* methodology section, where it currently appears as the expanded Table 2 on page 22. Exhibit H-E (which summarizes the size of the companies in the "Gray Zone" and "High-Financial-Risk" zone for each of the eight alternative size measures) was also moved, and now appears in the High-Financial-Risk Study methodology section as Table 10 on page 111.

Also in 2011, our Design team gave the *Risk Premium Report* a fresh new look that features a "double column" format that is easier to read, and saves paper.

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[5]   A free copy of the unlevered Exhibit C premia for the *2010 Risk Premium Report* is available at www.DuffandPhelps.com/CostofCapital

How the 2013 Report is Organized

The *Risk Premium Report* is divided into two main sections: a methodology section, followed by a data exhibits section.

Section 1: Methodology

The first section features a discussion of the data and methodology used to create the portfolios, which are the focus of the analysis in the *Report*, as well as an overview of the *Size Study*, *Risk Study*, and *High-Financial-Risk Study* (with examples of how to use each of these studies to estimate cost of equity capital). This is followed by a section on properly using the "C" Exhibits to further refine cost of equity capital (COE). Appendices, a Glossary of terms, and a "frequently asked questions" (FAQ) section are also included:

- **Portfolio Methodology:** A discussion of the data and methodology used to create the portfolios, which are the focus of the analysis in the *Report.*

- **Size Study:** Analyzes the relationship between equity returns and company size, using up to eight measures of company size (i.e. "size measures").

- **Risk Study:** Analyzes the relationship between equity returns and accounting-based fundamental risk measures.

- **High-Financial-Risk Study:** Analyzes the relationship between equity returns and high-financial-risk, as measured by the Altman z-Score.

- **C Exhibits:** The C Exhibits can help *Report* users to further refine their COE estimates by comparing their subject company's fundamental risk factors to the fundamental risk factors of the companies that comprise the 25 *Size Study* portfolios.

- **Appendices:** Definitions of *Compustat* data items (Appendix A); summary of changes from previous versions of the *Report* (Appendix B); Overview of Duff & Phelps' recommended U.S. ERP and matching risk-free rate (Appendix C).

- **Glossary:** A list of important terms with accompanying definitions.

- **FAQ:** Answers to some of the most frequently asked questions about the *Report.*

How the 2013 Report
is Organized

Section 2: Data Exhibits

The second section describes the data exhibits in which the various risk and size premia used to estimate cost of equity capital are found.

Each of the three *Studies* (*Size Study*, *Risk Study*, and *High-Financial-Risk Study*) discussed in the Methodology section have corresponding data Exhibits (A, B, D, or H), as illustrated in Figure 1.

Figure 1: *Size Study, Risk Study, High-Financial-Risk Study* **and Corresponding Exhibits**
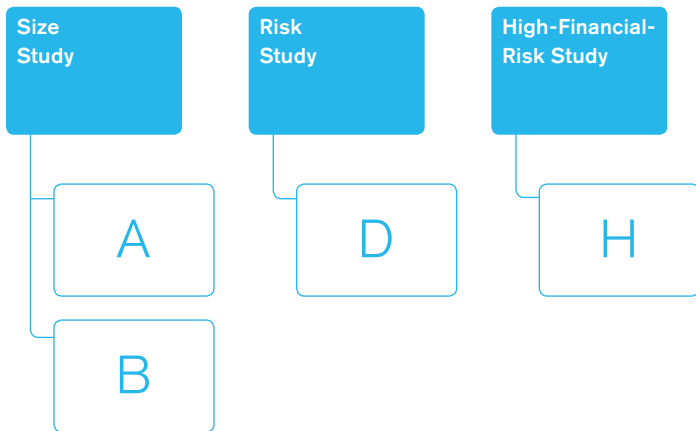
The risk premia and size premia reported in the A, B, D, and H exhibits can be used to develop cost of equity capital estimates using both the buildup method and the capital asset pricing model (CAPM). In addition, the C exhibits provide a "link" between the 25 size-ranked portfolios in the *Size Study*'s A and B exhibits and the three accounting-based fundamental risk characteristics used in the *Risk Study* (see page 112 for a full discussion of the proper use of the C exhibits).

- **Exhibits A-1 through A-8:** The A exhibits provide risk premia over the risk-free rate in terms of the combined effect of *market risk* and *size risk* for 25 portfolios ranked by eight alternative measures of size (RP_{m+s}).

- **Exhibits B-1 through B-8:** The B exhibits provide risk premia over CAPM ("size premia") in terms of *size risk* for 25 portfolios ranked by eight alternative measures of size (RP_s).

- **Exhibits C-1 through C-8:** The C exhibits provide a "link" between the 25 size-ranked portfolios in the *Size Study*'s A and B exhibits and the three accounting-based fundamental risk characteristics used in the *Risk Study*. These exhibits can be used to compare a subject company's fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of similarly-sized companies.

For example, the C exhibits can help to answer whether the subject company is more or less profitable (as measured by operating margin) than similarly-sized companies, or whether the subject company's earnings are more or less volatile (as measured by coefficient of variation of operating margin and coefficient of variation of ROE) than similarly-sized companies.

In the former case (which is a measure of profitability), the less profitable the subject company is, all other things held the same, the riskier it is (and vice versa). In the latter two cases (which are measures of earnings volatility), the more volatile a company's earnings are, all other things held the same, the less predictable they are, and thus the riskier the company is (and vice versa).

This is an important capability because this type of analysis can be used as an indication as to whether an upward or downward adjustment to a risk premium or size premium (and thus, COE) might be justified, based upon the so-called "company-specific" differences of the subject company' fundamental risk relative to the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.

Figure 2: The C Exhibits – *A "Link" Between the Size Study Portfolios and Accounting-Based Fundamental Risk Characteristics*
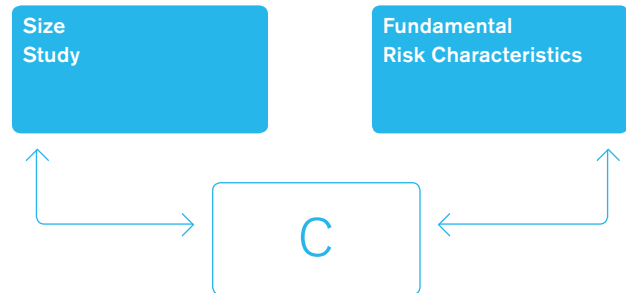
- **Exhibits D-1, D-2, and D-3:** The D exhibits provide risk premia over the risk-free rate in terms of the combined effect of *market risk* and *company-specific risk*, as represented by the differences in three alternative accounting-based measures of fundamental risk (RP_{m+u}).

- **Exhibits H-A, H-B, and H-C:** The H exhibits provide "high-financial-risk" premia for portfolios ranked by Altman z-Score.[6] These premia may be used in both buildup and CAPM estimates of cost of equity capital if the individual analyst has determined that the subject company is considered "high-financial-risk".[7] Exhibit H-A is the high-financial-risk equivalent of the A exhibits, Exhibit H-B is the high-financial-risk equivalent of the B exhibits, and Exhibit H-C is the high-financial-risk equivalent of the C exhibits.

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[6] Altman z-Score is an accounting-data-based method designed to assess financial condition and developed originally for assessing the likelihood of bankruptcy.

[7] The decision to apply a high-financial-risk premium is ultimately dependent on the analyst's professional judgment, based upon the analyst's detailed knowledge of the subject company.

Portfolio Methodology

Data Sources

The universe of companies used to perform the analyses presented in the *Risk Premium Report* is comprised of those companies that are found in both the Center for Research in Security Prices (CRSP) database at the University of Chicago Booth School of Business *and* Standard and Poor's *Compustat* database.

Historical Time Period Used

In the *2013 Risk Premium Report*, risk premia and other useful statistics are developed using historical equity returns (from CRSP), and fundamental accounting data (from *Compustat*) over the period 1963 through 2012.

The *Compustat* database was established in 1963. While *Compustat*'s fundamental accounting data is available for some companies going back to the 1950s, this earlier data consists only of the back-filled histories (5 years prior to 1963) for companies that were added to *Compustat* in 1963 or later. The *Report*'s analysis begins with 1963 data in order to avoid the obvious selection bias that would result from using the earlier data.

For each year covered in the *Report*, financial data for the fiscal year ending no later than September of the previous year is considered. For example, when assigning a company to a portfolio to calculate returns for calendar year 1995, financial data through the latest fiscal year ending September 1994 or earlier is considered (depending on when the company's fiscal year ended).

Exclusions

After identifying a universe of companies that are in both the CRSP and *Compustat* databases, the following types of firms are excluded:

- American Depository Receipts (ADRs)
- Non-operating holding companies
- Financial service companies (SIC code 6)

Financial service companies (those companies in finance, insurance, or real estate) are excluded because some of the financial data used in the *Report* is difficult to apply to companies in the financial sector (for instance, "sales" at a commercial bank). In addition, financial service companies tend to support a much higher ratio of debt to equity than do other industries, and so including them in with non-financial firms may be an "apples to oranges" comparison that could lead to improperly skewed results. Moreover, companies in the financial services sector were poorly represented during the early years of the *Compustat* database.

It should be noted that since financial service companies are excluded from the set of companies used to perform the analyses presented in the *Report*, these results should not be used by an analyst estimating the cost of equity capital (COE) for a financial services company.

Altogether, companies are excluded (or segregated) in the *Risk Premium Report* based upon their past financial performance or trading history. It should be noted that alternative analyses in which *no* companies were excluded or segregated on the basis of past financial performance or trading history have been performed (that is, using all available non-financial companies). The results are similar, but these exclusions are maintained as a precaution against the possibility of introducing a bias in favor of the size effect (to the extent that such companies tend to have low market values).

Portfolio Methodology

Unseasoned Companies

The small cap universe may consist of a disproportionate number of start-up companies and recent initial public offerings. These "unseasoned" companies may be inherently riskier than companies with a track record of viable performance. For this reason (for each year since 1963), we screen the universe of companies to exclude companies with any of the following characteristics:[8]

- Companies lacking 5 years of publicly traded price history

- Companies with sales below $1 million in any of the previous five fiscal years

- Companies with a negative 5-year-average EBITDA (earnings before interest, taxes, depreciation and amortization) for the previous five fiscal years

- Companies not listed on one of the major US stock exchanges (NYSE, NYSE MKT[9] or NASDAQ)

The set of companies remaining after this screen are seasoned companies in that they have been traded for several years, have been selling at least a minimal quantity of product or services, and have been able to achieve a degree of positive cash flow from operations.

High-Financial-Risk Study

After eliminating companies with the characteristics described previously, the remaining companies are screened again to exclude companies with any of the following characteristics:[10]

- Companies that Standard & Poor's has identified in the *Compustat* database as in bankruptcy or in liquidation.

- Companies with a "5-year average net income available to common equity" less than zero for the previous five years (either in absolute terms or as a percentage of the book value of common equity).

- Companies with "5-year-average operating income" (net sales minus cost of goods sold; selling, general and administrative expenses; and depreciation) less than zero for the previous five years (either in absolute terms or as a percentage of net sales).

- Companies with negative book value of equity at any one of the company's previous five fiscal year-ends.

- Companies with a debt-to-total capital ratio exceeding 80%, (debt is measured in book value terms, and total capital is measured as book value of debt *plus* market value of equity).

The companies excluded in this screen are set aside and analyzed separately in the *High-Financial-Risk Study*.

This screen is performed in an effort to isolate the effects of high-financial-risk. Otherwise, the results might be biased for smaller companies to the extent that highly leveraged and financially distressed companies tend to have both erratic returns and low market values.

It is possible to imagine companies that don't have any of these characteristics, but could *still* be classified as high-financial-risk (i.e. "distressed"), and it is also possible to imagine companies which do have one or more of these characteristics, but are *not* distressed. Nevertheless, the resulting high-financial-risk database is composed largely of companies whose financial condition is significantly inferior to the average, financially "healthy" public company.

Exclusions are Based on Past Information

The exclusion of companies is based on their *past* financial performance or trading history as of the time that the portfolios are formed for any given year over the 1963–2012 time horizon. For example, to form portfolios for 1963, company data for the previous 5 fiscal years (prior to September 1962) is considered. This procedure is repeated for *each* year from 1963 through the latest available year for *each* of the eight measures of size examined in the *Size Study*, and for *each* of the three measures of fundamental risk examined in the *Risk Study*. All of the previously discussed exclusions are therefore not based on any unusual foresight on the part of hypothetical investors in these portfolios, but are based on information that was already "history" at the time the portfolios were created.

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[8]   The number of companies eliminated in this screen varies from year to year.

[9]   The "NYSE MKT" is the former American Stock Exchange, or AMEX. On October 1, 2008, NYSE Euronext acquired the American Stock Exchange (AMEX). Post merger, the AMEX equities business was branded "NYSE Alternext US". NYSE Alternext US was subsequently re-branded "NYSE Amex Equities". On May 10, 2012, NYSE Euronext (NYX) announced that NYSE Amex LLC will be renamed NYSE MKT LLC effective May 14, 2012, subject to SEC filing.

[10] The number of companies eliminated in this screen varies from year to year. These companies represented up to 25% of the data set in recent years, but less than 5% in 1963. Certain technical changes in methodology have resulted in a greater number of companies falling into the high-financial-risk database than in versions of this study published prior to 2000.

Portfolio Methodology

Portfolio Creation

After excluding unseasoned and segregating high-financial-risk companies, the result is a base set of companies that is used for the analyses performed in both the *Size Study* and the *Risk Study*.

The major difference between the two studies is that the portfolios presented in the *Size Study* are ranked by eight alternative measures of size, from largest (Portfolio 1) to smallest (Portfolio 25), while the portfolios presented in the *Risk Study* are ranked by three accounting-based measures of fundamental risk, from lowest risk (Portfolio 1) to highest risk (Portfolio 25). The smallest size/highest risk portfolios tend to have the highest returns.

Other than that difference, portfolio formation in the *Size Study* and *Risk Study* is a very straightforward process. This process is described in the following sections.

Size Study Portfolio Creation

To perform the analysis required for the *Size Study*, 25 portfolios are created from companies that are similarly-sized, with Portfolio 1 made up of the largest companies and Portfolio 25 made up of the smallest companies. The equity returns for each of the 25 portfolios returns are calculated using an equal-weighted average of the companies in the portfolio, and these returns are then used to calculate risk premia (and other useful information and statistics) for each.

Equal-weighted rather than market-cap-weighted returns are used in the formation of both the *Size Study* and *Risk Study* portfolios because one could consider the former type of returns most relevant to the analyst applying the results of the *Risk Premium Report* in practice.

Specifically, our analysis of equal-weighted returns mimics the approach of the analyst matching his or her subject company with a group of comparable firms based on a set of risk characteristics that includes size and financial risk. As the returns of each firm in the group of comparable firms are potentially equally informative about the effect of matching (risk) characteristics on the cost of equity, the equal-weighted realized returns of the peer group are likely more relevant to the analysis than are the market-cap-weighted returns. Using equal-weighted returns is also analogous to analysts using equal-weighted valuation multiples in comparables valuation methods.[11]

"Size" is defined by the traditional size measure, market value of common equity (i.e. market capitalization), as well as seven additional size measures:

1) Market value of common equity
2) Book value of common equity
3) 5-year average net income
4) Market value of invested capital (MVIC)
5) Total assets
6) 5-year average EBITDA[12]
7) Sales
8) Number of employees

The first step is to determine portfolio breakpoints for the 25 portfolios. Portfolio breakpoints are the upper and lower "boundaries" of each portfolio, represented by the largest and smallest New York Stock Exchange (NYSE) company, respectively, in each of the 25 portfolios. For example, to determine the breakpoints for the 25 portfolios ranked by "Total Assets", all of the companies in the base set that are traded on the NYSE are ranked from largest (in total assets) to smallest (in total assets), and then divided into 25 equally populated portfolios.

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[11] As described in *"European Risk Premium Report"*, Erik Peek, Rotterdam School of Management, Erasmus University, working paper, January 2013.
[12] Earnings before interest, income taxes, depreciation and amortization.

Portfolio Methodology

Once portfolio breakpoints are determined, companies from the NYSE MKT (formerly the AMEX) universe and the NASDAQ universe are added to the appropriate portfolio, depending on their size with respect to the breakpoints.[13] Since NYSE MKT and NASDAQ companies are generally small relative to NYSE companies, their addition to the data set produces portfolios that are more heavily populated at the "small cap" end of the spectrum.[14]

All portfolios are rebalanced annually, so this process is completed for *each* year from 1963 to the most recent available year, and for *each* of the eight measures of size. This results in the creation of 25 portfolios for each of the eight size measures, a total of 200 (8 x 25) unique portfolios for each year from 1963 to present, each ranked from largest to smallest by each respective size measure.[15]

Risk Study Portfolio Creation

To perform the analysis required for the *Risk Study*, 25 portfolios are created from companies that have similar accounting-data-based fundamental risk characteristics, with Portfolio 1 made up of companies with the lowest fundamental risk, and Portfolio 25 made up of companies with the highest fundamental risk.

The returns for each of the 25 portfolios are calculated using an equal-weighted average of the companies in the portfolio, and these returns are then used to calculate risk premia (and other useful information and statistics) for each.

"Fundamental Risk" is defined by the following three alternative measures (the first is a measure of profitability; the latter two are measures of earnings variability):

1) Operating margin

2) Coefficient of variation in operating margin

3) Coefficient of variation in return on equity

As in the *Size Study*, the first step is to determine portfolio breakpoints for the 25 portfolios. Using "Operating Margin" as an example, all companies in the base set that are traded on the New York Stock Exchange (NYSE) are ranked from lowest fundamental risk (highest operating margin) to highest fundamental risk (lowest operating margin), and then divided into 25 equally populated portfolios.

Once portfolio breakpoints are determined, companies from the NYSE MKT universe and the NASDAQ universe are added to the appropriate portfolio, depending on their fundamental risk with respect to the breakpoints.

Since all portfolios are rebalanced annually, this process is followed for *each* year from 1963 to the most recent available year, for *each* of the three measures of fundamental risk. This results in the creation of 25 portfolios for each of the three fundamental risk measures, a total of 75 (3 x 25) unique portfolios for each year from 1963 to present, each ranked from lowest risk to highest risk for each respective measure of fundamental risk.[16]

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[13] NYSE MKT (formerly the AMEX) data is available after 1962 and NASDAQ data is available after 1972.

[14] Some readers may ask why NYSE breakpoints are used rather than ranking the entire NYSE/NYSE MKT/NASDAQ universe. The consistent use of NYSE breakpoints avoids an apples-to-oranges mixing of pre-1972 (pre-NASDAQ) ranking criteria with post-1972 ranking criteria. Otherwise, "average" NASDAQ companies (in recent years) would be assigned to portfolios that contain much larger "average" NYSE companies (in earlier years) when calculating average returns for the mid-sized portfolios over the full sample period. The only logical alternatives are either to adopt the NYSE breakpoint approach or to exclude NASDAQ companies altogether.

[15] In the *2013 Report*, this represents 8 size measures x 25 portfolios x 50 years (1963–2012) = 10,000 unique portfolio formations to perform the analysis presented in the *Size Study*.

[16] In the *2013 Report*, this represents 3 measures of fundamental risk x 25 portfolios x 50 years (1963–2012) = 3,750 unique portfolio formations to perform the analysis presented in the *Risk Study*.

Portfolio Methodology

Correcting for Delisting Bias

Previous evidence indicated that the CRSP database omits delisting returns for a large number of companies for the month in which a company is delisted from an exchange.[17] Data was collected for a large number of companies that had been delisted for performance reasons (e.g. bankruptcy, or insufficient capital) and found that investors incurred an average loss of about 30% after delisting.

While CRSP has improved its database by reducing the number of companies for which it omits delisting returns, we incorporate this evidence into our rate of return calculations by applying a 30% loss in the month of delisting in all cases where the delisting return is missing and for which CRSP identified the reason for delisting as "performance related". As an additional precaution, this adjustment is also applied in all cases in which the reason for delisting was identified by CRSP as "unknown".[18]

Size and Risk Rankings are Based on Past Information

The ranking of companies based on size and fundamental risk does not imply any unusual foresight on the part of hypothetical investors in these portfolios – the data used is as of the *beginning* of each year, and thus was already "history" at the time the portfolios are formed.

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[17] "The Delisting Bias in CRSP Data," Tyler Shumway, *Journal of Finance* (March 1997).

[18] This approach is consistent with updates that we have published since 1998. More recent evidence suggests that the average "delisting" loss is less than Shumway's original estimate. For more information about CRSP and CRSP delisting returns, visit www.CRSP.com

Using the 2013 Report

Choosing Inputs when Estimating the Cost of Equity Capital

When estimating cost of equity capital (COE) by applying the *Duff & Phelps Risk Premium Report*, the user typically starts by making a few basic choices: a risk-free rate (R_f), an equity risk premium (ERP) and risk premium over CAPM from the B exhibits (i.e., "size premium") (RP_s), or a risk premium over the risk-free rate from the A or the D exhibits (RP_{m+s}). Depending on the method selected to estimate COE, the ERP Adjustment may also have to be applied to account for the difference between the forward-looking ERP selected for use in the COE calculations, and the historical (1963–present) ERP used as a *convention* in the *Report*.

Key Inputs for Estimating the Cost of Equity Capital

The Risk-Free Rate: A risk-free rate (R_f) is the expected return on a security that the market generally regards as free of the risk of default.[19] Like all cost of capital components, the risk free rate is "expected" (i.e., forward-looking), and the term of the risk-free asset should generally match the expected life of the investment being valued. For example, when valuing a business for which a "going concern" assumption can reasonably be made, many analysts will select the 20-year (constant-maturity) U.S. Treasury yield as of the valuation date as a proxy for the risk-free rate.[20, 21, 22]

Because investors expect that the returns from an investment in equities to be *at least as much* as the returns that they would receive from an investment in a risk-free asset, most of the widely used methods for estimating the cost of equity capital (e.g., build-up method, CAPM and Fama-French three-factor model) *begin* with the yield to maturity on U.S. government securities (as of the valuation date), and *then* build upon that.

The Equity Risk Premium (ERP): The equity risk premium (often interchangeably referred to as the market risk premium) is defined as the incremental return over the expected yield on risk-free securities (typically U.S. Treasury bonds) that investors expect to receive from an investment in a diversified portfolio of common stocks.[23] Like all other COE inputs, the equity risk premium is forward looking. The equity risk premium is a key input used in most methods for estimating the COE, including both of the methods used in the *Risk Premium Report* (the buildup method and the CAPM). There is no *single* universally accepted

methodology for estimating the ERP, and consequently there is wide diversity in practice among academics and financial advisors with regards to recommended ERP estimates.

The Equity Risk Premium (ERP) Adjustment: The ERP Adjustment accounts for the difference between the forward-looking ERP selected as of the valuation date for use in the COE calculation, and the historical (1963–present) ERP used as a *convention* to create the *Report*. The size premium over the risk-free rate (RP_{m+s}) *always* requires application of the ERP Adjustment; whereas the size premium over CAPM (RP_s) (i.e., size premium) *never* requires application of the ERP Adjustment because it is already reflected in the CAPM estimate. For a detailed discussion of the ERP Adjustment, see page 12.

Refer to Table 1 on page 14 for a complete listing of all COE estimation methods available in the Duff & Phelps Risk Premium Report, and whether or not the ERP Adjustment is necessary for each.

Risk premium over the risk-free rate (RP_{m+s}): This premium reflects risk in terms of the combined effect of *market* risk and size risk in excess of the risk-free rate. This premium can be added to a risk-free rate (R_f) to estimate cost of equity capital in a "buildup" method, and is found in the A, C, and D exhibits. Risk premium over the risk-free rate (RP_{m+s}) *always* requires application of the ERP Adjustment.

Risk Premium Over CAPM or Size premium (RP_s): This premium reflects the return in excess of what CAPM predicts, and is otherwise known as the *size premium* (RP_s). This premium can be added to a CAPM-based cost of equity estimate as an adjustment for size, and is found in the B exhibits. A risk premium over CAPM *never* requires application of the ERP Adjustment because again it is already captured in CAPM.

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[19] An alternative definition of a risk-free asset is an asset for which the investor knows the expected future economic benefits with certainty.

[20] To be precise, long-term U.S. government bonds are not entirely "risk-free." For example, bond prices are sensitive to future interest rate fluctuations. Also, investors do not know what (future) rate will be available for reinvesting coupon payments (this is sometimes referred to as "reinvestment" risk).

[21] Note that the "expected life of the investment" may be quite different from the "holding period" of any particular investor.

[22] There are several sources of risk-free rates, but some of the most common may include the Wall Street Journal, Bloomberg, or the Federal Reserve Board of Governors' website at www.federalreserve.gov/releases/h15/data.htm

[23] Shannon Pratt and Roger Grabowski, *Cost of Capital: Applications and Examples* 4th ed. (New York; John Wiley & Sons, 2010), page 115.

Using the 2013 Report

Proper Application of the Equity Risk Premium (ERP) Adjustment

Some users of the *Duff & Phelps Risk Premium Report* may not be aware of the equity risk premium (ERP) Adjustment.[24] In this section, the following topics are discussed:

- The ERP Adjustment Defined

- Calculating the ERP Adjustment

- When the ERP Adjustment is (and is not) Necessary

- A Step-By-Step Example of the ERP Adjustment

The ERP Adjustment Defined

The ERP Adjustment is needed to account for the difference between the forward-looking ERP as of the valuation date that a *Report* user has selected to use in his or her cost of equity capital calculations, and the historical (1963–present) ERP that was used as a *convention* in the calculations performed to create the *Report*.[25, 26] In other words, if a *Report* user's estimate of the ERP for the S&P 500 on a forward-looking basis is materially different from the historical ERP as measured over the time horizon 1963–present, it is reasonable to assume that the *other* historical portfolio returns reported here would differ on a forward-looking basis by a similar amount. The ERP Adjustment accounts for this differential.

Some may ask why the historical 1963–present ERP is used as the convention in the calculations performed to produce the *Report*. The short answer is that choosing the historical ERP calculated over the same time horizon that corresponds to the accounting and return data available from the CRSP and *Compustat* databases seems a natural choice. Also, it would be quite impractical to recalculate and publish the *Report* using every conceivable ERP that an individual valuation professional might select – there is a wide diversity in practice among academics and financial advisors with regards to ERP estimates, and we recognize in a practical sense that there is *also* a wide diversity of ERP estimates used by financial professionals in valuation engagements. So, a single ERP is selected to use as a convention to calculate the *Report*'s risk premia and size premia, and the individual analyst adjusts accordingly, given his or her selected ERP as of the valuation date.

Calculating the ERP Adjustment

The ERP Adjustment is calculated as the simple difference between the ERP the *Report* user has selected for use in his or her cost of equity capital estimates minus the historical 1963–present ERP. In the *2013 Report* the historical ERP used as a convention in the calculations was 4.5 percent.[27] The ERP Adjustment for users of the *2013 Report* is thus calculated as follows:

ERP Adjustment = ERP that Report User has selected for use in COE estimates − Historical ERP (1963–2012)

ERP Adjustment = ERP that Report User has selected for use in COE estimates − 4.5%

Using the 2013 Report

When the ERP Adjustment is (and is not) Necessary

The ERP Adjustment is only applicable in specific cases, as outlined below.

To understand when the ERP Adjustment is (and is not) necessary, it is first important to understand that there are two types of premia published in the *Duff & Phelps Report*.[28] The first type of premia is "risk premia over the risk-free rate" (i.e., RP_{m+s}), which are a measure of risk in terms of the combined effect of *market risk* and *size risk*. Risk premia over the risk-free rate are simply added to a risk-free rate to estimate cost of equity capital (COE) if you are using the buildup method. The second type of premia published in the *Report* is "risk premia over CAPM" (i.e., RP_s), which are commonly referred to as "beta adjusted size premia", or simply "size premia", and are a measure of *size risk*. Size premia can be added as a size adjustment if you are using the capital asset pricing model (CAPM).

Because the premia published in the *Report* are historical averages measured over the time horizon 1963–present, the historical ERP over the 1963–present time horizon for the S&P 500 Index[29] becomes embedded in the first type of risk premia, "risk premia over the risk-free rate" (RP_{m+s}). However, the historical ERP over the 1963–present time horizon does *not* become embedded in the second type of premia published in the *Report*, size premia (RP_s), because size premia are beta adjusted (i.e., market risk adjusted).[30] For this reason, there are only two basic ideas to remember in regards to when the ERP Adjustment is necessary:

- The ERP Adjustment is *always* necessary when using one of the *Report*'s "risk premia over the risk-free rate" (RP_{m+s}), because the historical ERP over the 1963–present time horizon for the S&P 500 Index is *embedded* in these premia. The *Report*'s risk premia over the risk-free rate (RP_{m+s}) come from Exhibits A-1 through A-8 (or from Appendix H-A, the "high-financial-risk" equivalent of the A exhibits), Exhibits C-1 through C-8, and Exhibits D-1 though D-3.

- The ERP Adjustment is *never* necessary when using one of the *Report*'s "size premia", because the historical ERP over the 1963–present time horizon does not become embedded in size premia since size premia are beta adjusted (i.e., market risk adjusted). The *Report*'s size premia come from Exhibits B-1 through B-8 (or from Appendix H-B, the "high-financial-risk" equivalent of the B exhibits).

The decision process is very straightforward, and is outlined in Figure 3:

Figure 3: Is the ERP Adjustment Necessary?
Decision Tree

| Are you using a "Risk Premium Over the Risk Free Rate" (RP_{m+s}) or a "Size Premium" (RP_s)? |

RP_{m+s} → Calculate the ERP Adjustment and add it to your COE estimate

RP_s → Do not add the ERP Adjustment to your COE estimate

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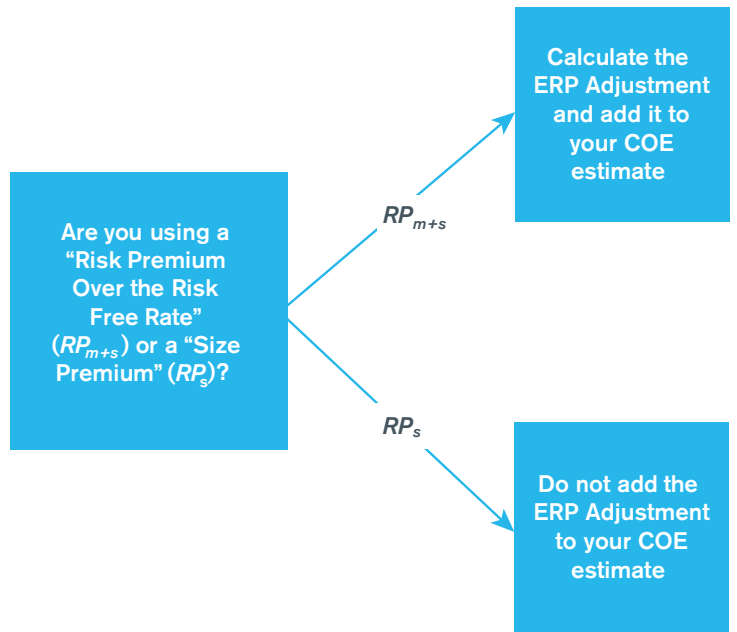
[28] For a detailed discussion of the two types of premia published in the *Report*, see *"The Difference Between 'Risk Premia Over the Risk-Free Rate' and 'Risk Premia Over CAPM'"* on page 43.

[29] The "historical" ERP was 4.5% over the period 1963-2012.

[30] A "beta-adjusted" size premia has been adjusted to remove the portion of excess return that is attributable to beta, leaving only the size effect's contribution to excess return.

Using the 2013 Report

Table 1 lists all of the methods available in the *Risk Premium Report* to calculate the cost of equity capital (COE), and the equations for each. This table is very useful in that it provides a complete list of the methods available in the *Risk Premium Report* to estimate COE, clearly identifies which of the methods require an ERP Adjustment (and which methods do not), and also provides the source of the various premia used in each of the models.[31, 32]

Note that in Table 1 the "Buildup 1" method and the "CAPM" method are highlighted. These two methods are probably the most commonly used methods of estimating COE using the *Risk Premium Report*. So, in many cases, the question of whether the ERP Adjustment is necessary reduces to a question of whether the Buildup 1 method is being used, which utilizes a "risk premium over the risk-free rate" (RP_{m+s}), and *always* requires an ERP Adjustment, or the CAPM method is being used, which utilizes a "size premium" (RP_s), and *never* requires an ERP Adjustment.

Table 1: All COE Estimation Methods Available in the *Duff & Phelps Risk Premium Report*

Report Study	*Method*	*Equation*	*Source of Premium*	*ERP Adjustment?*
Size Study	Buildup 1	$COE_{subject\ company} = R_f + RP_{m+s}$ $+ ERP\ Adjustment$	A Exhibits	Yes
Size Study	Buildup 1-Unlevered	$COE_{subject\ company} = R_f + RP_{m+s,\ unlevered}$ $+ ERP\ Adjustment$	C Exhibits	Yes
Size Study	CAPM	$COE_{subject\ company} = R_f + (ß \times ERP) + RP_s$	B Exhibits	No
Size Study	Buildup 2	$COE_{subject\ company} = R_f + ERP$ $+ RP_s + IRP_{Adj}$	B Exhibits	No
Risk Study	Buildup 3	$COE_{subject\ company} = R_f + RP_{m+u}$ $+ ERP\ Adjustment$	D Exhibits	Yes
Risk Study	Buildup 3-Unlevered	$COE_{subject\ company} = R_f + RP_{m+u,\ unlevered}$ $+ ERP\ Adjustment$	D Exhibits	Yes
High-Financial-Risk Study	Buildup 1-High-Financial-Risk	$COE_{subject\ company} = R_f + RP_{m+s,\ high-financial-risk}$ $+ ERP\ Adjustment$	H-A Exhibits	Yes
High-Financial-Risk Study	CAPM-High-Financial-Risk	$COE_{subject\ company} = R_f + (ß \times ERP)$ $+ RP_{s,\ high-financial-risk}$	H-B Exhibits	No

[31] The *Risk Premium Report* provides two ways for users to match their subject company's size (or risk) characteristics with the appropriate smoothed premia: the "guideline portfolio" method, and the "regression equation" method. The equations shown in Table 1 are valid for both the guideline portfolio method and the regression equation method. To learn more about the guideline portfolio method and the regression equation method, see page 19.

[32] In Table 1, the "Buildup 2" method incorporates an adjusted "Industry Risk Premium" (*IRP_{Adj}*). Duff & Phelps does not publish Industry Risk Premia. Industry Risk Premia are available in Table 3-5 of the *2013 SBBI Valuation Yearbook* (Morningstar, Chicago), page 32. See "Estimating Cost of Equity Capital Using the 'Buildup 2' Method" on page 76.

Using the 2013 Report

A Step-By-Step Example of the ERP Adjustment

Calculating the ERP Adjustment is straightforward. The following example uses data from the *2013 Report*, and additional information about prior versions of the *Report* is included for completeness and convenience.

Step 1: Identify the historical 1963–present ERP used as a convention in the calculations performed to create the *Report*. The historical market risk premiums that were used in the calculations to create the last five *Risk Premium Reports* (from the *2009 Report* to the *2013 Report*) are shown in Figure 4.[33]

Figure 4: Historical Market Risk Premiums Used in Risk Premium Report Calculations
2009 Report–2013 Report

Report Year	Historical Period Used in *Report* Calculations	Historical ERP as of *Report* Version
2013 Risk Premium Report	1963–2012	4.5%
2012 Risk Premium Report	1963–2011	4.3%
2011 Risk Premium Report	1963–2010	4.4%
2010 Risk Premium Report	1963–2009	4.3%
2009 Risk Premium Report	1963–2008	3.9%

Looking to Figure 4, the historical ERP that was used as a convention in the calculations performed to create the *2013 Report* is 4.5 percent. If the analyst has selected, say, the Duff & Phelps Recommended ERP [34] as of December 31, 2012 (5.5%) as the ERP to use in his or her COE calculations, the ERP Adjustment is 1.0 percent:

ERP Adjustment = ERP selected for use in COE estimates − Historical ERP (1963–2012)

1.0% = 5.5% − 4.5%

This implies that on a forward-looking basis as of the valuation date, investors expected to earn 1.0 percent more than they realized on average over the period 1963–2012.

If the analyst had instead selected, say, the long-term "historical" ERP of 6.7 percent as calculated over the time period 1926–2012[35] to use in his or her COE calculations, the ERP Adjustment would then be 2.2 percent:

ERP Adjustment = ERP selected for use in COE estimates − Historical ERP (1963–2012)

2.2% = 6.7% − 4.5%

This implies that on a forward-looking basis as of the valuation date, investors expected to earn 2.2 percent more than they realized on average over the period 1963–2012.

Step 2: Determine if the ERP Adjustment is necessary by looking at Table 1 on page 14. Probably the easiest way to determine this is to look at the fourth column in Table 1, "Source of Premium". Which exhibit did the premium used in the COE estimate come from? For example, if one is using the Buildup 1 method, then the "risk premium over the risk-free rate" was found in the "A" exhibits. In this case, as noted in the fifth column of Table 1, the ERP Adjustment *needs to be added* to the COE estimate:

$$COE_{subject\ company} = R_f + RP_{m+s} + ERP\ Adjustment$$

Alternatively, if one is using the CAPM method, then the "risk premium over CAPM" (i.e., size premium) was found in the "B" exhibits. In this case, as noted in the fifth column of Table 1, the ERP Adjustment *does not need to be added* to the COE estimate:

$$COE_{subject\ company} = R_f + (\beta * ERP) + RP_s$$

Of course, the same decision process can be used for any of the other methods of estimating COE available in the *Duff & Phelps Risk Premium Report* and listed in Table 1. For example, if one were using the Buildup 2 method, which utilizes a size premium (RP_s) rather than a risk premium over the risk-free rate (RP_{m+s}), then the ERP Adjustment does not need to be added to the COE estimate:

$$COE_{subject\ company} = R_f + ERP + RP_s + IRP_{Adj}$$

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[33] The historical ERP employed in the calculations performed to create the *Risk Premium Report* is derived by subtracting the annual average income return of *SBBI* long-term government Treasury bonds from the average annual total return of the S&P 500 Index. Source: Morningstar *EnCorr* software.

[34] See page 128 for a detailed discussion of the Duff & Phelps Recommended ERP.

[35] Calculated by Duff & Phelps and derived by subtracting the annual average income return of SBBI long-term government Treasury bonds from the average annual total return of the S&P 500 Index. Source of underlying data: Morningstar *EnCorr* software.

Using the 2013 Report

One final note: What if the ERP Adjustment is *not* made to the models in Table 1 that indicate that the adjustment is necessary? In cases where the ERP Adjustment is not applied (as indicated in Table 1 on page 14), the net effect is that the historical 1963–present ERP used in the calculations to create the *Report* is embedded in the COE estimate. By doing this, the *Report* user is in effect adopting the historical ERP as measured over the 1963–present time period as his or her forward-looking ERP, *which may or may not be the ERP that the user wishes to use as of his or her valuation date*.

For example, the ERP used as a convention in the calculations to create the *2013 Risk Premium Report* was the historical 1963–2012 market risk premium (4.5%). If the *Report* user estimates COE using the "Buildup 1" method, which requires an ERP Adjustment, the ERP embedded in his or her estimate is 4.5% even though it is not "visible" in the equation. If in the same valuation engagement the *Report* user then estimates COE using CAPM and selects a 5.5% ERP to use in the CAPM equation, the two models are now not in harmony: two *different* ERPs have effectively been used in the same engagement (4.5% in the case of the Buildup 1 estimate, and 5.5% in the case of the CAPM estimate). The way to bring them back into harmony is simply to always apply the ERP Adjustment as indicated in Table 1.

Using the 2013 Report

Using "Smoothed" Premia versus Using "Average" Premia

The difference between average risk premia and smoothed risk premia is illustrated in Graph 1(a) and Graph 1(b).

Graph 1(a): Average Risk Premia for 25 Portfolios with a Best Fit Line Added

A scatter plot of risk premia smoothed in this fashion and the log of the size measures will necessarily fall on the best fit line (smoothed risk premia are represented by the blue diamonds in Graph 1(b)).

Graph 1(b): Smoothed Risk Premia
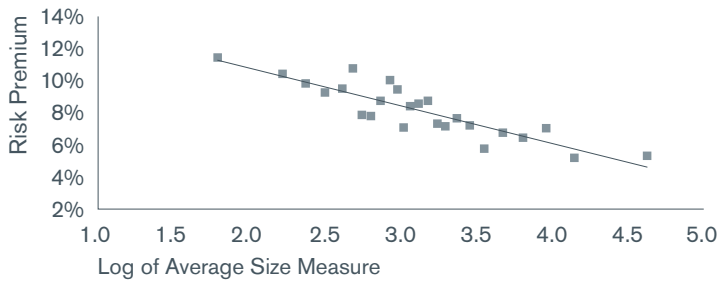
In Graph 1(a), the square gray points represent a scatter plot of size (on the horizontal "x" axis), and the average risk premium (for each of 25 size-ranked portfolios, on the vertical "y" axis).[36] Note that as size increases from left to right, the risk premium tends to decrease (and vice versa).

The "best fit" line is the straight ("smooth") line in Graph 1(a). Using regression analysis, an equation for the best fit line can be calculated, and this equation can be used to estimate "smoothed" risk premia for the 25 portfolios based upon the average size measure of each portfolio.

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[36] In this example, "risk premium" is used generically. The same statistical techniques described in this example are used to calculate smoothed "risk premia over the risk-free rate" (the A exhibits) and "risk premia over CAPM" (the B exhibits), as well as smoothed unlevered premia (the C exhibits).

Using the 2013 Report

Smoothing the premia essentially averages out the somewhat scattered nature of the raw average premia. The "smoothed" average risk premium is generally the most appropriate indicator for most of the portfolio groups. It should be noted, however, that at the largest-size and smallest-size ends of the range, the average historical risk premiums may tend to jump off of the smoothed line, particularly for the portfolios ranked by size measures that incorporate market capitalization (Exhibits A-1 and A-4). Because the size measure is expressed in logarithms, this is equivalent to the change in risk premium given the percentage change in the size of the companies from portfolio to portfolio.

Smoothed risk premia are found in the data exhibits. For example, in Figure 5 the smoothed average risk premium over the risk-free rate for Portfolio 24 in Exhibit A-3 is 11.86%, and the smoothed average risk premium over the risk-free rate for Portfolio 25 is 13.14%.[37]

In this example, the 11.86 percent smoothed average risk premium is calculated based upon the average 5-year average net income of companies in Portfolio 24 ($11 million). However, the subject company's size rarely exactly matches the average size of companies in the guideline portfolio. In the next section, how to interpolate an "exact" risk premium value when the subject company's size is "in between" guideline portfolios is explained.

Figure 5: Smoothed Premia in Exhibit A-3

Companies Ranked by 5-Year Average Net Income
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Net Income (in $millions)	Log of Average Net Income	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Smoothed Average Risk Premium	Average Debt/ MVIC
1	7,844	3.89	38	0.77	15.95%	11.65%	12.87%	6.11%	3.92%	20.20%
2	2,243	3.35	28	0.87	15.63%	10.60%	11.77%	5.02%	5.44%	24.84%
3	1,412	3.15	31	0.86	15.97%	12.12%	13.31%	6.56%	6.00%	26.82%
←				///						→
24	11	1.06	114	1.25	24.59%	15.52%	18.21%	11.46%	11.86%	24.32%
25	4	0.60	265	1.32	29.22%	17.34%	21.14%	14.39%	13.14%	25.42%

[37] The A Exhibits include "risk premia over the risk-free rate" which are added to a risk-free rate to estimate cost of equity capital using the buildup method. Please refer to the individual examples provided for these models for more information and examples.

Using the 2013 Report

Understanding the Difference Between the Risk Premium Report's "Guideline Portfolio Method" and "Regression Equation Method"

The *Risk Premium Report* provides two ways for valuation professionals to match their subject company's size characteristics with the appropriate smoothed premium to be used to estimate the cost of equity capital: the "guideline portfolio" method and the "regression equation" method.

The major difference between the guideline portfolio method and the regression equation method is illustrated in Figure 6 (a) and 6(b), which are abbreviated portions of Portfolio 24 and Portfolio 25 from Exhibit A-3 from the *2013 Risk Premium Report*. Exhibit A-3 consists of 25 "guideline" portfolios sorted from largest (Portfolio 1) to smallest (Portfolio 25) according to 5-year average net income (Portfolios 3 through 23 are not shown in Figure 6).

Figure 6(a) represents the guideline portfolio method. In Figure 6(a), the average net income of Portfolio 24 is $11 million, and the average net income of Portfolio 25 is $4 million. If one were using the guideline portfolio method and your subject company (Subject Company A) had net income of say, $7 million, you would select Portfolio 25's *published* risk premium of 13.14%, because $7 million is *closer* to $4 million than it is to $11 million.

Alternatively, if your subject company (Subject Company B) had a net income of say, $2 million, you would *again* select Portfolio 25's *published* risk premium of 13.14%, because $2 million is *closest* to Portfolio 25's average net income of $4 million.

The regression equation method, however, allows the valuation professional to calculate an *interpolated* risk premia "in between" portfolios, and also to calculate *interpolated* risk premia for companies with size characteristics less than the average size in Portfolio 25.

Figure 6(b) represents the regression equation method. In Figure 6(b), Subject Company A's net income of $7 million falls *in between* the average net income of Portfolio 24 ($11 million) and Portfolio 25 ($4 million), and the regression equation method enables the valuation professional to calculate an exact interpolated risk premium of 12.45%, which is *in between* the *published* risk premia of Portfolio 24 (11.86%) and Portfolio 25 (13.14%). In the second case, Subject Company B's net income of $2 million is *smaller* than the average size of $4 in Portfolio 25, and an exact interpolated risk premium of 13.98% is calculated, which is *higher* than the 13.14% risk premium of Portfolio 25.

These results are intuitive – as size *decreases*, risk premia (and thus cost of equity capital) tend to *increase*. In the next section, an example of how to calculate custom "interpolated" values using the *Risk Premium Report's* regression equation method is provided.

Companies Ranked by 5-Year Average Net Income
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Figure 6(a): Guideline Portfolio Method

Portfolio Rank By Size	Average Net Inc. (in $millions)	Smoothed Average Risk Premium
1	7,844	3.92%
2	2,243	5.44%
///	///	///
24	11	11.86%

Subject Company A → ⑦

25 ④ → *13.14%*

Subject Company B → ②
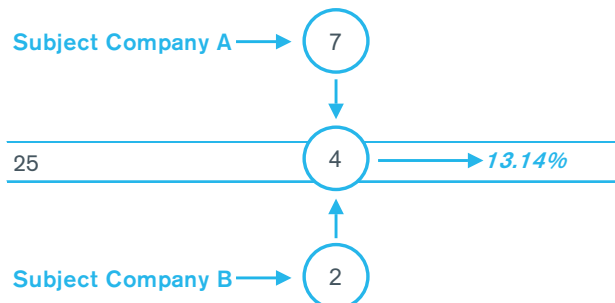
Figure 6(b): Regression Equation Method

Portfolio Rank By Size	Average Net Inc. (in $millions)	Smoothed Average Risk Premium
1	7,844	3.92%
2	2,243	5.44%
///	///	///
24	11	11.86%

Subject Company A → ⑦ → *12.45%*
(Interpolated)

25 4 *13.14%*

Subject Company B → ② → *13.98%*
(Interpolated)
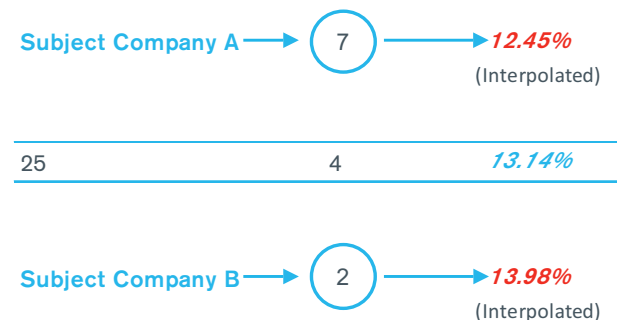
Using the 2013 Report

Example: Calculating an Interpolated Premium Using the Regression Equation Method

In almost all cases, the subject company's size will not exactly match the average size characteristics of the guideline portfolio (in the *Risk Premium Report*) or the decile (in the *SBBI Yearbook*). The *Risk Premium Report* provides the regression equation method, which is a straightforward and easy way to calculate an interpolated premium "in between" portfolios based upon the subject company' specific size characteristics, rather than simply accepting the published premium associated with the average size of the selected guideline portfolio.

A quick refresher of the regression equation method in the *Risk Premium Report* is outlined in this section. Then, a discussion (and guidance) is provided on what to do when the size characteristics of

subject company are less than the characteristics of the average size of the companies that comprise the *Risk Premium Report's* 25th portfolios.

In the *Risk Premium Report*, regression equations are provided for each data exhibit. Figure 7 (which displays Exhibit A-3 from the *2013 Duff & Phelps Report*), has 25 portfolios ranked by the size measure "Five-Year Average Net Income".[38] Net income is one of the eight different measurements of size provided in the *Risk Premium Report*. Exhibits A-1 through A-8 are used to estimate cost of equity capital using the "buildup" method under each of these eight size measures. Alternatively, Exhibits B-1 through B-8 are used to estimate cost of equity capital using the capital asset pricing model, or CAPM for the same size measures.

Figure 7: Exhibit A-3 from the *2013 Duff & Phelps Report*
Companies Ranked by Five-Year Average Net Income

Exhibit A-3

Companies Ranked by 5-Year Average Net Income
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Premia Over the Risk-Free Rate (RP_{m+s})

Equity Risk Premium Study: Data through December 31, 2012
Data Smoothing with Regression Analysis
Dependent Variable: Average Premium
Independent Variable: Log of Average Net Income

Portfolio Rank by Size	Average Net Inc. (in millions)	Log of Average Net Inc.	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	*Smoothed Average Risk Premium*	Average Debt/ MVIC
1	7,844	3.89	38	0.77	15.95%	11.65%	12.87%	6.11%	*3.92%*	20.20%
2	2,243	3.35	28	0.87	15.63%	10.60%	11.77%	5.02%	*5.44%*	24.84%
3	1,412	3.15	31	0.86	15.97%	12.12%	13.31%	6.56%	*6.00%*	26.82%
4										
5										
6										
7										
8										
9	276	2.44	38	1.05	18.23%	12.75%	14.34%	7.59%	*7.99%*	23.64%
10	227	2.36	33	1.07	17.96%	12.56%	14.05%	7.30%	*8.23%*	23.99%
11	196	2.29	37	1.08	19.78%	13.84%	15.62%	8.87%	*8.40%*	23.62%
12	172	2.24	29	1.08	20.24%	13.86%	15.78%	9.03%	*8.56%*	24.45%
13	152	2.18	34	1.06	18.81%	14.32%	15.96%	9.21%	*8.72%*	23.90%
14	128	2.11	39	1.15	20.10%	12.69%	14.63%	7.88%	*8.92%*	24.11%
15	108	2.03	45	1.12	21.90%	12.89%	15.06%	8.30%	*9.13%*	23.65%
16	97	1.99	40	1.13	20.08%	14.73%	16.61%	9.86%	*9.26%*	23.28%
17	83	1.92	43	1.18	21.13%	13.44%	15.54%	8.79%	*9.45%*	22.72%
18	71						17.08%	10.33%	*9.64%*	23.
19	61						16.00%	9.24%	*9.83%*	23.
20	49						18.29%	11.54%	*10.10%*	24.
21	35						17.25%	10.50%	*10.49%*	24.
22	25						16.87%	10.12%	*10.88%*	24.
23	18	1.25	107	1.26	24.54%	16.15%	18.74%	11.99%	*11.31%*	.21%
24	11	1.06	114	1.25	24.59%	15.52%	18.21%	11.46%	*11.86%*	24.32%
25	4	0.60	265	1.32	29.22%	17.34%	21.14%	14.39%	*13.14%*	25.42%

Regression Equation

Smoothed Premium = 14.818% - 2.798% * Log(Net Income)

Portfolio	Avg. Net Inc
24	$11
25	$4

Smoothed Premium
11.9%
13.1%

	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium
Large Stocks (Ibbotson SBBI data)		9.80%	11.21%	4.46%
Small Stocks (Ibbotson SBBI data)		13.44%	16.17%	9.42%
Long-Term Treasury Income (Ibbotson SBBI data)	6.73%	6.75%	

Regression Output:

Constant	14.818%
Std Err of Y Est	0.864%
R Squared	86%
No. of Observations	25
Degrees of Freedom	23
X Coefficient(s)	-2.798%
Std Err of Coef.	0.236%
t-Statistic	-11.87

Smoothed Premium = 14.818% - 2.798% * Log(Net Income)

Smoothed Premium vs. Unadjusted Average

Log of Average 5-year Average Net Income

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[38] Portfolio 1 is comprised of the *largest* companies; Portfolio 25 is comprised of the *smallest* companies.

Using the 2013 Report

Each of the data exhibits In the *Risk Premium Report* includes a regression equation (it is in the same place on each exhibit, as shown in Figure 7). The average net income of the companies comprising in Portfolio 24 is $11 million, and the average net income of the companies comprising Portfolio 25 is $4 million. If you were using the guideline portfolio method and your subject company had net income of say, $7 million, you would select Portfolio 25's risk premium of 13.1% (rounded) when employing the buildup method because $7 million is closer to $4 million than it is to $11 million.

But the subject company's net income of $7 million is *in between* the average net income of Portfolios 24 and 25, and you may prefer to calculate an *interpolated* premium amount rather than simply accepting the *published* 13.1% premium indicated by the average net income of companies comprising Portfolio 25. This is easy to do using the "regression equation" method – just substitute the net income of your subject company (in this case, "7") for "net income" into the regression equation listed on the exhibit:[39]

= 14.818% - 2.798% * LOG(net income)

= 14.818% - 2.798% * LOG(7)

= 12.5%

Portfolio 24's *published* premium is 11.9% (rounded), Portfolio 25's *published* premium is 13.1%, and the calculated *interpolated* premium based on the subject company's net income of $7 million is *in between* these two premiums, at 12.5%. If we are using a normalized risk-free rate of 4.0%, then the cost of equity capital using the "Buildup 1" method for the subject company is:[40]

Cost of equity capital = risk-free rate + interpolated premium from Exhibit A-3

Cost of equity capital = 4% + 12.5%

Cost of equity capital = 16.5%

Tips Regarding the Regression Equation Method

- The regression equations are *different* for each exhibit.
- Estimate cost of equity capital using as many of the eight "A" exhibits (A-1 through A-8) that are available for the subject company, and then calculate the average and median of all of these estimates. This principle is also true when using the "B" exhibits (B-1 through B-8) to estimate cost of equity capital using the CAPM.
- Never use those size measures for which the subject company's size measure is equal to zero or negative.

- The online *Risk Premium Calculator* automatically applies the *Risk Premium Report* data for the subject company using both the "guideline portfolio method" and the "regression equation method" for each of the size characteristics selected.

Can the Regression Equation Method be Used If the Subject Company is Small?

The previous example was for a subject company whose size characteristics were greater than the average company in Portfolio 25. Can we apply the *Risk Premium Report* data for a subject company whose size characteristics are *less* than the average company included in Portfolio 25?[41]

The short answer is "Yes". It may be appropriate to extrapolate the risk premium for companies whose size characteristics are less than the average characteristics of the companies comprising the bottom half of Portfolio 25 using the regression equation method. While extrapolating a statistical relationship far beyond the range of data used in analysis is generally not recommended, in cost of capital analyses (or any analysis for that matter), there is always the question of "compared to what?" Put simply, while it may not be ideal to extrapolate a statistical relationship beyond a certain range, one may be confronted with a situation in which no better measure is available.

In order to assess the appropriateness of using the regression equations for small companies, Table 2 is provided in the *2013 Risk Premium Report*.[42] This table can help the valuation professional gauge the size characteristics of the subject company relative to the size characteristics of companies that comprise Portfolio 25 for each of the eight size measures, and provide support for the additional adjustment provided by the regression equations.

Specifically, in cases where the size characteristic of the subject company is significantly less than that of the average company included in Portfolio 25 for any given size measure, the valuation professional may report the individual, average and the median premia (and corresponding cost of equity capital estimates) using both the guideline portfolio method and the regression equation method (using all of the subject company size characteristics that are available). However, we recommend that the valuation professional consider disclosing that the subject company's selected size metric is less than, for instance, the smallest of companies included in Portfolio 25 of a particular size measure. Once again, reporting all of the information in a transparent way is preferable to not reporting it at all, especially in cases where no better alternative is available.

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[39] In this equation, rather than using the traditional multiplication symbol "x," we put the equation into the Microsoft Excel format by using the character "*". Also, remember to enter the amount in "millions" (e.g., $7,000,000 should be entered as 7).

[40] This is the cost of equity capital estimate using five-year net income as the measure of size only. Duff & Phelps recommends using as many of the eight size measures as possible for best results.

[41] Portfolio 25 is comprised of the *smallest* companies; Portfolio 1 is comprised of the *largest* companies.

[42] The information in the *2013 Risk Premium Report's* "Table 2" describing the size characteristics of Portfolio 25 for each of the eight size measures was initially published in the *2009 and 2010 Duff & Phelps Reports* as "Exhibit E,", then as "Table 1" in the *2011 Duff & Phelps Report*, then as Table 5 in the *2012 Report*. Also note that the size values in this table *change* each year when the *Duff & Phelps Report* is updated.

Using the 2013 Report

Using Table 2 from the 2013 Risk Premium Report

The *2013 Risk Premium Report* summarizes the size characteristics for Portfolio 25 for *each* of the eight size measures in both the "A" exhibits and the eight "B" exhibits, as shown in Table 2. While the "A" and "B" exhibits present *different* types of risk premia, the "A" and "B" exhibits are:

• Comprised of the same set of companies, and

• Ranked by the same eight alternative measures of size.

This is done for a specific reason: It ensures that the "risk premia over the risk free rate" that are published in the "A" exhibits and used in the buildup method are "apples to apples" when compared to the "risk premia over CAPM" (also known as "size premia") that are published in the "B" exhibits.

Table 2: Size Measures of Companies that Comprise Portfolio 25, by Percentile
(in $ millions, except for Number of Employees)

	Market Value of Equity	Book Value of Equity	5-year Average Income	Market Value of Invested Capital
Smallest Company	$1.222	$4.327	**$0.190**	$2.031
5th Percentile	14.038	10.877	**0.453**	16.871
25th Percentile	39.855	28.409	1.853	50.012
50th Percentile	82.089	58.803	3.627	106.092
75th Percentile	145.941	99.105	6.165	182.843
95th Percentile	203.438	137.708	**8.270**	275.641
Largest Company	219.936	149.763	**8.816**	285.930

	Total Assets	5-year Average EBITDA	Sales	Number of Employees
Smallest Company	$5.791	$0.317	$1.671	3
5th Percentile	17.682	1.814	16.517	11
25th Percentile	50.984	6.126	52.578	114
50th Percentile	104.899	13.698	102.842	233
75th Percentile	179.849	24.081	166.343	377
95th Percentile	255.632	34.120	248.753	517
Largest Company	274.802	36.979	266.356	560

The use of this table is straightforward. For example, the "95th Percentile" of size in Portfolio 25 for "5-year Average Net Income" is $8.270 million, which means that 95% of the companies in Portfolio 25 have average income that is *less* than $8.270 million (alternatively,

this means that 5% of the companies in Portfolio 25 have average income that is *greater* than $8.270 million). Or, looking to the 5th percentile, 5% of the companies in Portfolio 25 have average income that is *less* than $0.453 million (alternatively, this means that 95% of the companies in Portfolio 25 have average income that is *greater* than $0.453 million).

To provide even greater detail about the composition of the portfolios used to calculate the premia in the *Risk Premium Report*, starting with the *2013 Report* Table 2 also includes the *smallest* and *largest* companies in the 25th portfolio of each of the eight size measures. For example, the largest company in Portfolio 25 for "5-year Average Net Income" is $8.816 million. Alternatively, the smallest company in Portfolio 25 for "5-year Average Net Income" is $0.190 million.

When using the *Risk Premium Report* to estimate cost of equity capital for a subject company, three cases are possible:

• **Case 1:** *All* of the subject company's size characteristics are *greater* than the smallest companies in Portfolio 25 of *each* of the eight size measures reported in the *2013 Risk Premium Report's* Table 2.

In Case 1, the valuation professional can report the individual, average and median premia (and corresponding cost of equity capital estimates) using both the guideline portfolio method and calculated using the regression equation method (using all of the subject company size characteristics that are available).[43]

• **Case 2:** *Some* of the subject company's size characteristics are *greater* than the smallest companies in Table 2, and some of the subject company's size metrics are *less* than the smallest companies in Table 2.

In Case 2, the valuation professional may consider reporting the individual, average and median premia (and corresponding cost of equity capital estimates) using both the guideline portfolio method and calculated using the regression equation method (using all of the subject company size characteristics that are available) segregated in three different groupings:

• The premia calculated using subject company size characteristics that are *greater* than the smallest company;

• The premia calculated using subject company size characteristics that are *less* than the smallest company; and

• A combined grouping of *all* the premia calculated using subject company size characteristics (both those *greater* than, and *less* than, the smallest company).

Using the 2013 Report

In many cases, the difference between the three groupings' concluded cost of equity capital estimates will be small.

- **Case 3:** *None* of the subject company's size characteristics are *greater* than the smallest companies as reported in Table 2.

In Case 3, the valuation professional may report the individual, average and median premia (and corresponding cost of equity capital estimates) using both the guideline portfolio method and calculated using the regression equation method (using all of the subject company size characteristics that are available). We recommend that the valuation professional disclose that the subject company's size characteristics are less than the smallest companies included in Portfolio 25 for all size measures.

The Risk Premium Report's Regression Equation Method Yields Results that are Intuitive

In Graph 2, the regression equation method was used to calculate a cost of equity capital estimate across a broad range of each of the eight size measures of Portfolio 25 (a 4.0% *normalized* risk-free rate was used in this analysis). The medians of the eight cost of equity capital estimates for the smallest, largest, and average-sized companies are shown in blue, and the medians of the eight cost of equity capital estimates for the 95th, 75th, 50th, 25th, 5th percentiles are shown in gray. In addition, the median of the eight cost of equity

capital estimates for a size of one half the size of the 5th percentile (0.5 x 5th) is also shown (in gray).

It makes intuitive sense that as we move further to the right in Graph 2 (and size decreases *within* Portfolio 25), the median cost of equity capital estimate increases. For example, when the *largest* companies for each of the eight size measures are substituted into their corresponding regression equations, the median value of these eight cost of equity capital estimates is a little more than 15% (the first blue bar on the left in Graph 2). Continuing with this idea, when the *average-sized* companies (which are, of course, smaller than the "largest" companies) listed for each of the eight size measures are substituted into their corresponding regression equations, the resulting median cost of equity capital estimate is *higher*, at approximately 17% (the second blue bar from the left in Graph 2). Using the "average-sized" company is the same result one would get using the "guideline portfolio" method, where one accepts the premium calculated for the *published* "average" sized company within a portfolio.

Finally, when the smallest companies (as listed in Table 2) for each of the eight size measures are substituted into their corresponding regression equations, the resulting median cost of equity capital estimate is again higher, at approximately 21% (the rightmost blue bar in Graph 2).

Graph 2: Median Cost of Equity Capital (COE) Estimate Using All Eight of the *Risk Premium Report's* Measures of Size, Over Various Company Sizes *within* Portfolio 25.
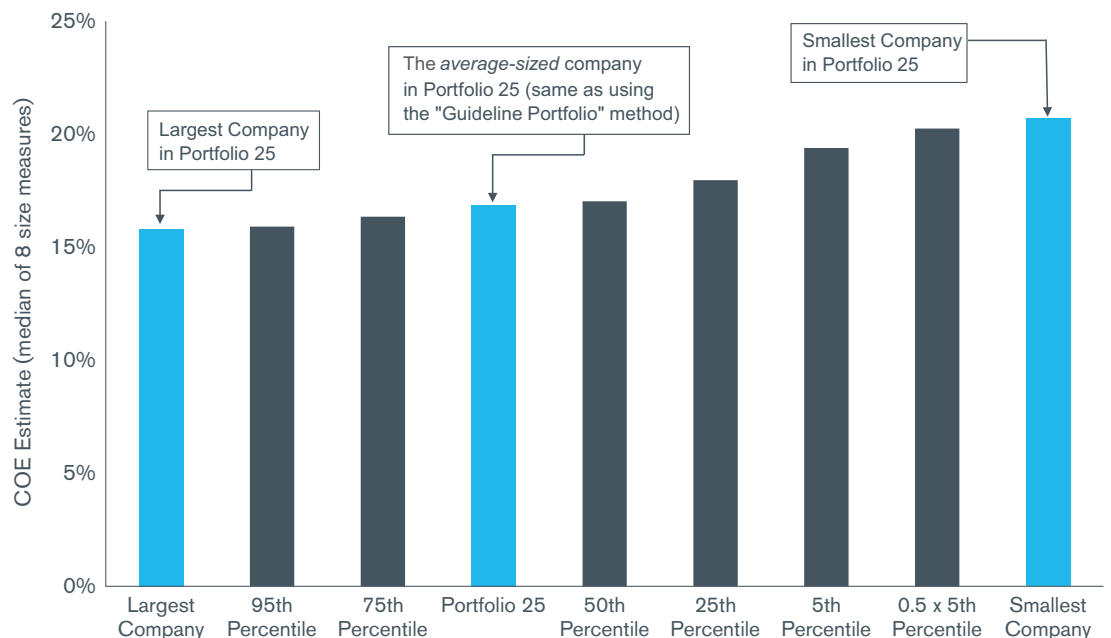
Using the 2013 Report

Further, if one examines the graphs in the "A" and "B" exhibits that display the smoothed (interpolated) premium compared to the unadjusted average premium, the interpolated premium is generally *less* than the unadjusted average for Portfolio 25, as shown in Graph 3.[44] This indicates that the extrapolated premium implies returns lower than the returns one might expect for very small companies, and is possibly a more conservative estimate.

In summary, when using the guideline portfolio method, the valuation professional matches the subject company's size characteristics to the guideline portfolio that has an average size *closest* to the subject company's size characteristics, and then uses the *published* smoothed premium published for that portfolio.[45]

Alternatively, the regression equation method provided in the *Risk Premium Report* is a natural extension of the guideline portfolio method, and allows the valuation professional to calculate custom *interpolated* risk premia for:

• "Risk premia over the risk-free rate" and "size premia" *in between* portfolios, and

• "Risk premia over the risk-free rate" and "size premia" *smaller* than the average company in the smallest portfolio.

Size Study or Risk Study?

Use both. Analysts can use the *Size Study* if it has been determined that the risks of the subject company are comparable to the average of the portfolio companies of comparable size (e.g., comparable operating margin). One can determine the relative risk characteristics by looking at Exhibits C-1 through C-8.

But, we do not know precisely how the market prices risk. The *Risk Study* provides returns based on risk measures regardless of size. One would likely expect that returns are greater for say, Portfolio 25, in the size measured portfolios rather than Portfolio 25 in the risk measured portfolio because sometimes a large company has risk measures more like a small company, and vice versa. How much higher/lower should be the returns? The D exhibits may help identify the magnitude of the return adjustment (see pages 115 and 116 for examples of how to do this).

Graph 3: Interpolated Premium vs. Unadjusted (not Interpolated) Average Premium

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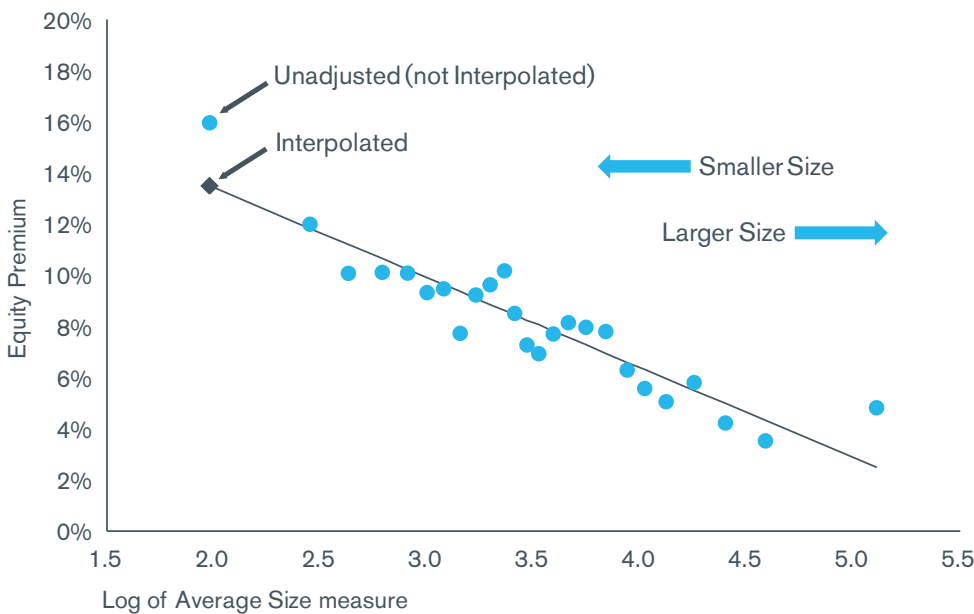
[44] Graph 3 is a generic example of the graphs published of adjusted versus unadjusted premia for each of the eight "A" exhibits (A-1 through A-8; these exhibits are used to estimate cost of equity capital using the buildup method), and for each of the eight "B" exhibits (B-1 through B-8; these exhibits are used to estimate cost of equity capital using the capital asset pricing model, or CAPM, method).

[45] In this way, the guideline portfolio method is akin to using the back page of the *SBBI Yearbook*, where the valuation professional matches the subject company's (estimated) market capitalization to the "decile" (i.e., portfolio) that has a comparable "market cap", and then uses the *published* size premium published for that decile.

The Size Study

The *Size Study* analyzes the relationship between equity (i.e., stock) returns and company size. In addition to presenting risk premia and size premia for 25 size-ranked portfolios using the traditional "market capitalization" measure, the *Size Study* also considers 7 additional measures of company size, including book value of equity, 5-year average net income, market value of invested capital (MVIC), total assets, 5-year average EBITDA, sales, and number of employees.[46] As demonstrated in Graph 4, the data shows a clear inverse relationship between size and historical rates of return, regardless of how size is measured.

In Graph 4, as size *decreases* (from left to right), the average annual return over the study time horizon (1963–2012) tends to *increase* for each of the eight size measures.

For example, in the *2013 Report*, the average annual return of the portfolios made up of the largest companies ("Portfolio 1" for each of the eight size measures) was 12.3 percent, while the average annual return of the portfolios made up of the smallest companies ("Portfolio 25" for each of the eight size measures) was 20.6 percent.

Moreover, the "size effect" is not just evident for the smallest companies, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of several billions of dollars.[47]

Graph 4: Average Annual Return, 8 Alternative Measures of Company Size
1963–2012

Legend:
- Market Value of Equity
- Book Value of Equity
- 5-year Average Net Income
- Market Value of Invested Capital (MVIC)
- Total Assets
- 5-year Average EBITDA
- Sales
- Number of Employees
- Average (all size measures)

Y-axis: Average Annual Return (5% to 25%)
X-axis: Portfolio (1 = Largest, 25 = Smallest)

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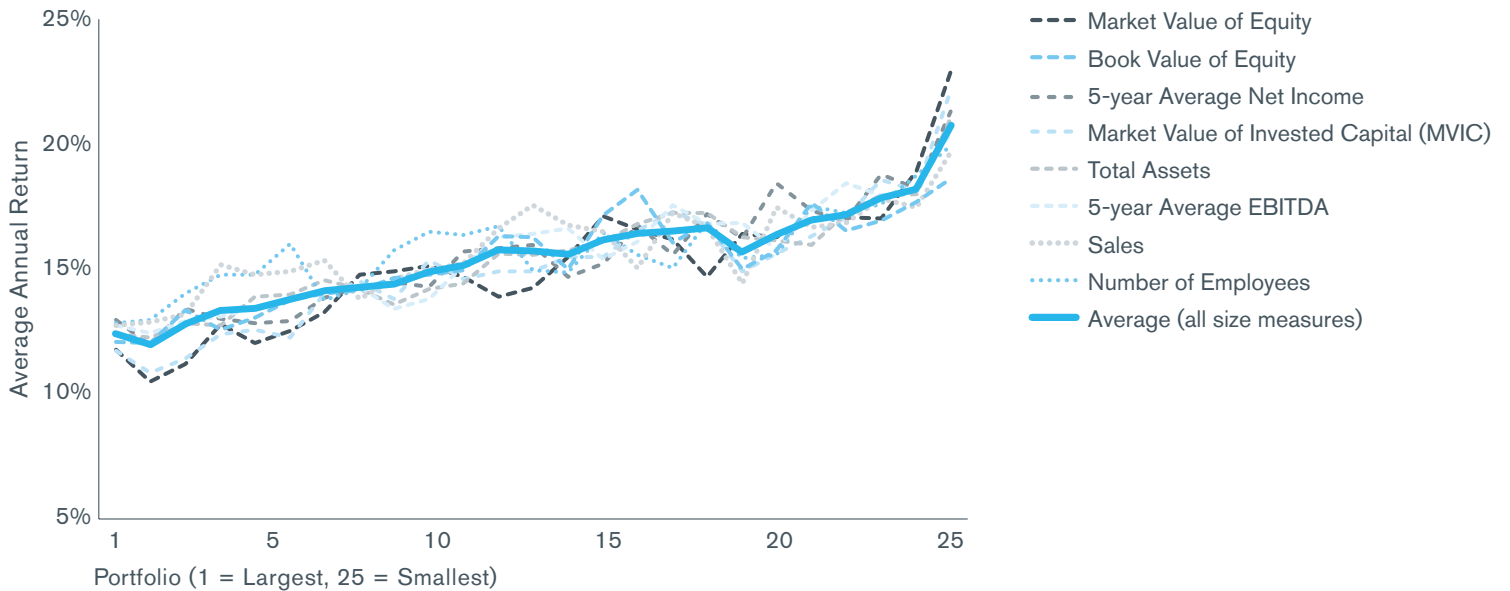
[46] For a detailed discussion of portfolio creation methodology, see "Portfolio Methodology" on page 6.

[47] While there is evidence of the size effect across the size spectrum, the size effect is not "linear". The effect is greatest in the smallest companies.

The Size Study

Reasons for Using Alternative Measures of Size

There are several reasons for using alternative measures of size in addition to market value of equity (i.e., "market capitalization" or simply "market cap"). First, financial literature indicates a bias may be introduced when ranking companies by market value because a company's market capitalization may be affected by characteristics of the company other than size.[48] In other words, some companies might be small because they are risky (high discount rate), rather than risky because they are small (low market capitalization). One simple example could be a company with a large asset base, but a small market capitalization as a result of high leverage or depressed earnings. Another example could be a company with large sales or operating income, but a small market capitalization due to being highly leveraged.

Second, market capitalization may be an imperfect measure of the risk of a company's operations.

Third, using alternative measures of size may have the practical benefit of removing the need to first make a "guesstimate" of size in order to know which portfolio's premium to use (this issue is commonly referred to as the "circularity" issue). When you are valuing a closely held company, you are trying to *determine* market capitalization. If you need to make a guesstimate of the subject company' market capitalization *first* in order to know which size premium to use, the "circularity" problem is introduced. While market capitalization, at least for a closely held firm, is *not* generally available, other size measures, such as assets or net income, *are* generally available.[49]

Finally, when doing analysis of any kind it is generally prudent to approach things from multiple directions if at all possible. This is good practice for several reasons, with the most important being that it has the potential of strengthening the conclusions of the analysis.

What is Size?

The size of a company is one of the most important risk elements to consider when developing cost of equity estimates for use in valuing a firm. Traditionally, researchers have used market value of equity as a measure of size in conducting historical rate of return research. For example, the Center for Research in Security Prices (CRSP) "deciles" are developed by sorting U.S. companies by market capitalization, and the returns of the Fama-French "Small minus Big" (SMB) series is the difference in return of "small" stocks minus "big" (i.e., large) stocks, as defined by market capitalization.[50, 51]

CRSP Databases

The creation of the CRSP databases at the University of Chicago in the early 1960s was a big advance in research in security prices. The CRSP database represents market value (stock price times the number of shares) and return data (dividends and change in stock price) going back to 1926. Prior to the creation of the CRSP databases, one literally had to gather data from old newspapers to do a retrospective valuation. However, possibly the most notable reason that the establishment of the CRSP databases was so critical was that it enabled researchers to look at stocks with different characteristics and analyze how their returns differed. With this capability we began to better understand the drivers of stock returns.

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[48] "A Critique of Size Related Anomalies," Jonathan Berk, *Review of Financial Studies*, vol. 8, no. 2 (1995).

[49] For further discussion of the history of the size premium and criticisms of the size premium, see Chapters 13 and 14 in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

[50] To learn more about the Center for Research in Security Prices (CRSP) at the University of Chicago Booth School of Business, visit www.CRSP.com

[51] Eugene Fama is the Robert R. McCormick Distinguished Service Professor of Finance at the University of Chicago, and Ken French is the Roth Family Distinguished Professor of Finance at the Tuck School of Business at Dartmouth College. Fama and French are prolific researchers and authors who have contributed greatly to the field of modern finance. Fama and French's paper "The Cross-Section of Expected Stock Returns" was the winner of the 1992 Smith Breeden Prize for the best paper in the *Journal of Finance*. Fama is also chairman of the Center for Research in Security Prices (CRSP) at the University of Chicago Booth School Of Business.

The Size Study

One of the characteristics that researchers first analyzed was large market capitalization (i.e., "large-cap") companies versus small market capitalization (i.e., "small-cap") companies. They divided the universe of publicly traded U.S. companies into 10 "deciles" (portfolios), with the largest-cap companies in Decile 1 and the smallest-cap companies in Decile 10. What they found was that the returns for small-cap companies were greater than the returns for larger-cap companies. In 1981, for example, a study by Rolf W. Banz examined the returns of New York Stock Exchange (NYSE) small-cap stocks compared to the returns of NYSE large-cap stocks over the period 1926–1975.[52] In Graph 5, the terminal index values of CRSP NYSE deciles 1–10 are shown as calculated over the same time period as Banz used in his 1981 study.[53] An investment of $1 at the end of 1925 in decile 1 (comprised of the largest-cap NYSE stocks) would have grown to $51 by the end of 1975, while an investment of $1 in decile 10 (comprised of the smallest-cap NYSE stocks) would have grown to $488 dollars by the end of 1975. Clearly, small-cap stocks exhibited significantly greater performance over this time period.

Graph 5: Terminal Index Values of CRSP NYSE Deciles 1–10 Index (Year-end 1925 = $1)
January 1926–December 1975

CRSP NYSE Decile
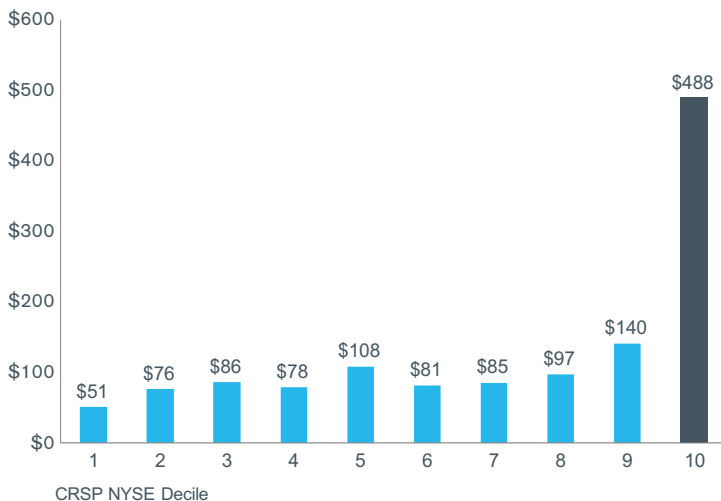
Possible Explanations for the Greater Returns of Smaller Companies

Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or another factor closely related to size. The qualification that Banz noted in 1981 remains pertinent today:

"It is not known whether size [as measured by market capitalization– ed.] per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size."

Practitioners know that small firms measured in terms of fundamental size measures such as assets or net income have risk characteristics that differ from those of large firms. For example, potential competitors can more easily enter the "real" market (market for the goods and/or services offered to customers) of the small firm and "take" the value that the small firm has built. Large companies have more resources to better adjust to competition and avoid distress in economic slowdowns. Small firms undertake less research and development and spend less on advertising than large firms, giving them less control over product demand and potential competition. Small firms have fewer resources to fend off competition and redirect themselves after changes in the market occur.[54] Smaller firms may have fewer analysts following them, and less information available about them. Smaller firms may have lesser access to capital, thinner management depth, greater dependency on a few large customers, and may be less liquid than their larger counterparts.[55] Each of these characteristics would tend to increase the rate of return that an investor might demand for investing in stocks of small companies rather than investing in stocks of large companies.

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[52] Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics* (March 1981): 3–18. Professor Banz's 1981 article is often cited as the first comprehensive study of the size effect.

[53] Calculated by Duff & Phelps based on CRSP® standard market-cap weighted NYSE decile returns. ©2013 Center for Research in Security Prices (CRSP®), University of Chicago Booth School of Business. Source: Morningstar EnCorr software.

[54] M. S. Long and J. Zhang, "Growth Options, Unwritten Call Discounts and Valuing Small Firms," EFA 2004 Maastricht Meetings Paper No. 4057, March 2004. Copy available at http:// ssrn.com/abstract=556203

[55] Even after controlling for size, research suggests that liquidity is still a predictor of return. See Roger G. Ibbotson, Zhiwu Chen, Daniel Y.-J. Kim, and Wendy Y. Hu, "Liquidity as an Investment Style", August 2012. SSRN id1817889. Copy available at www.zebracapital.com

The Size Study

Is the Size Effect Still Relevant?

Small-cap stocks do not always outperform large-cap stocks. For example, by one measure the worst performing 10-year period for small-cap stocks relative to large-cap stocks was the 10-year period ending March 1999.[56] Over this period large-cap stocks returned 515 percent, while small-cap stocks returned 162 percent, a difference of over 352 percent. Another example is the 10-year period ending July 1956, when large-cap stocks returned 349 percent and small-cap stocks returned 198 percent, a difference of 151 percent.

These examples alone do not nullify the size effect – if you believe that small companies are riskier than large companies, then it follows that small-cap stocks should not always outperform large-cap stocks in all periods.[57] By analogy, bond returns occasionally outperform stock returns, yet few would contend that over time the expected return on bonds is greater than the expected return on stocks.

However, the size effect is not immune to criticism. One commentator, for example, has stated that "…while the empirical evidence supports the notion that small cap stocks have earned greater returns after adjusting for beta risk than large cap stocks, it is not as conclusive, nor as clean as it was initially thought to be."[58]

The Size Effect Over Longer Time Periods

Small-cap stocks' outperformance of large-cap stocks appears to be a persistent trend over longer periods. For example, an investment of $1 at the end of 1925 in small-cap stocks would have grown to $42,347.63 by the end of 2012, while an investment of $1 at the end of 1925 in large-cap stocks would have grown to $1,951.66 (see Graph 6).[59]

Graph 6: Large-cap Stocks (CRSP Decile 1) and Small-cap Stocks (CRSP Decile 10)
Index (Year-end 1925 = $1)
January 1926–December 2012
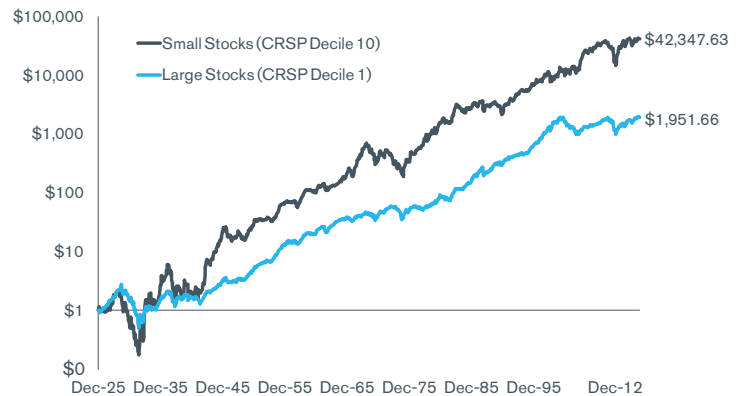
Small-cap stocks' shorter-term behavior relative to large-cap stocks can be especially erratic, so analyzing small-cap stocks' performance relative to large-cap stocks' performance over varying holding periods may be instructive in revealing longer-term trends. As the holding period is increased, the tendency of small-cap stocks to outperform large-cap stocks increases, as illustrated in Graphs 7(a), 7(b), 7(c), and 7(d).[60] In these graphs, the annual compound rate of return for large-cap stocks and small-cap stocks was calculated over all 5-, 10-, 20-, and 30-year periods from January 1926–December 2012.[61] The simple difference between small-cap stocks' returns and large-cap stocks' returns was then calculated for each period.

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[56] Derived by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. In this example, large-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 1; small-cap stocks represented by CRSP NYSE/AMEX/NASDAQ decile 10. Source: Morningstar *EnCorr Analyzer*. Note that we use the convention of referencing these series throughout the *Report* as currently named in the in the cited source (Morningstar EnCorr) as "NYSE/AMEX/NASDAQ", and not "NYSE/NYSE MKT/NASDAQ".

[57] Another way of stating this is if small company stocks <u>always</u> outperformed large company stocks, they would not be riskier than large company stocks.

[58] Aswath Damodaran, *"Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2011 Edition"*, Stern School of Business, February 2011, page 33.

[59] Derived by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. In this example, large-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 1; small-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 10. Source: Morningstar EnCorr software.

[60] Derived by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. In this example, large-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 1; small-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 10. Source: Morningstar EnCorr software.

[61] There are a total of 985 5-year (i.e, 60-month) periods, 925 10-year (i.e., 120-month) periods, 805 20-year (i.e., 240-month) periods, and 685 30-year (i.e., 360-month) periods, over the January 1926–December 2012 time horizon.

The Size Study

In Graph 7(a) small-cap stocks' returns were greater than large-cap stocks' returns in 57 percent of all 5-year (i.e., 60-month) periods ending December 1930 through December 2012. As the holding period is increased to 20 years and 30 years (see Graph 7(c) and 7(d) on the following page), small-cap stocks outperform large-cap stocks in a greater percentage of periods.

Graphs 7(a) and 7(b): Large-cap Stocks (CRSP Decile 1) versus Small-cap Stocks (CRSP Decile 10)
Difference in annual compound rates of return over 5- and 10-year holding periods.
January 1926–December 2012

Graph 7(a)
5-year periods
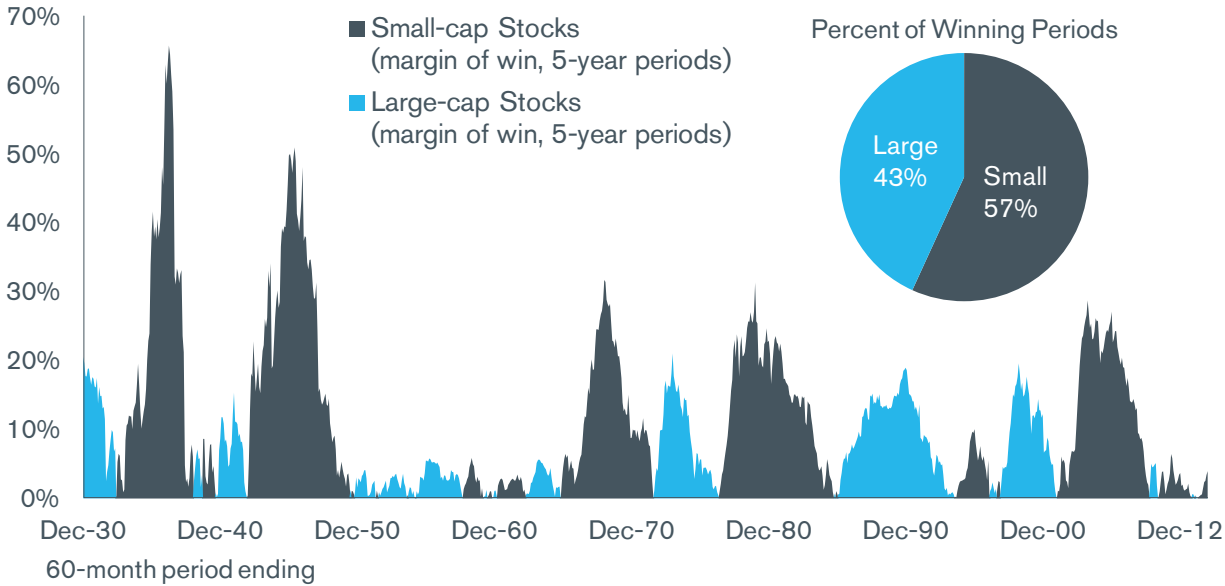
Graph 7(b)
10-year periods
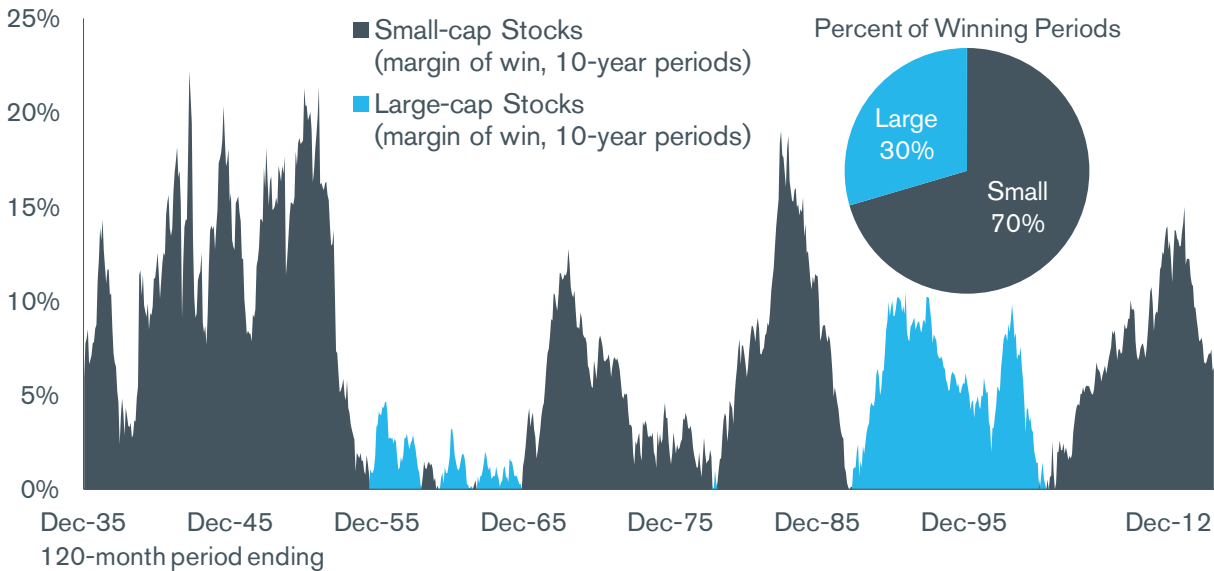
The Size Study

In Graph 7(d), for example, small-cap stocks' returns were greater than large-cap stocks' returns in 92 percent of all 30-year (i.e., 360-month) periods ending December 1955 through December 2012. Small-cap stocks outperformed large-cap stocks in nearly all 30-year periods, with the exception of 30-year periods ending in the late 1990s and early 2000s.

Graphs 7(c) and 7(d): Large-cap Stocks (CRSP Decile 1) versus Small-cap Stocks (CRSP Decile 10)
Difference in annual compound rates of return over 20- and 30-year holding periods.
January 1926–December 2012

Graph 7(c)
20-year periods
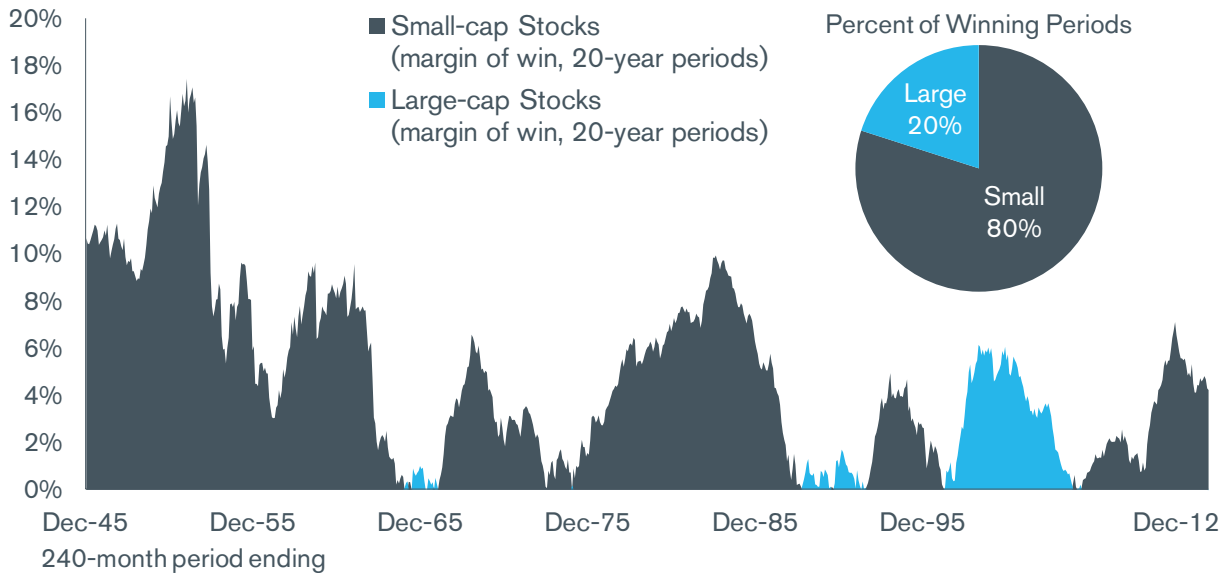
Graph 7(d)
30-year periods
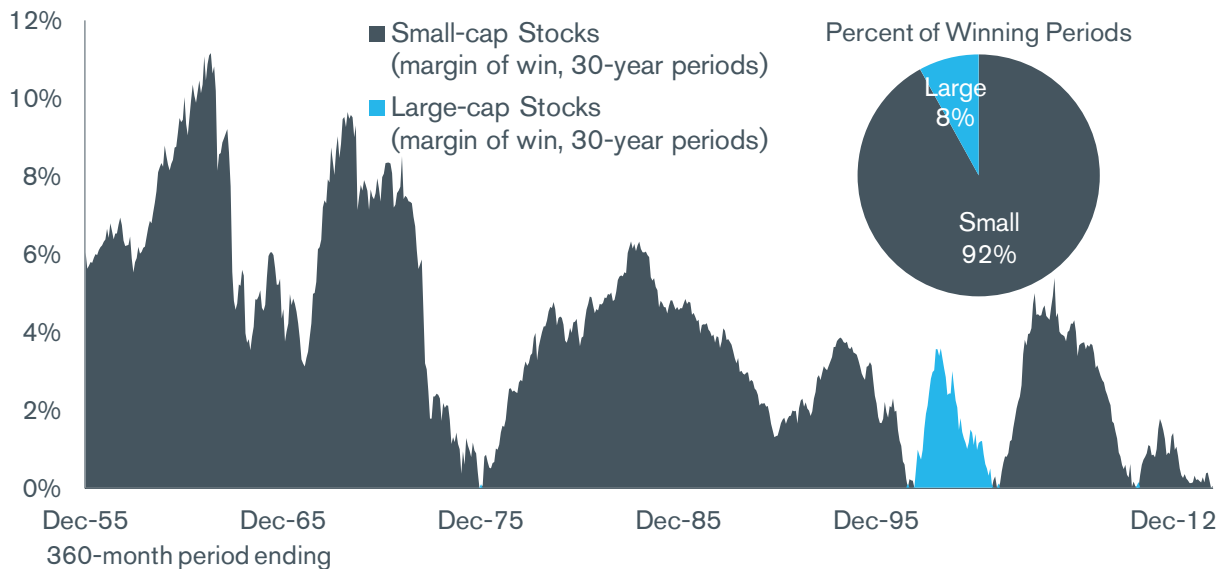
The Size Study

The Size Effect with Boom Years Omitted

Some research has suggested that if certain periods in which small-cap stocks greatly outperformed large-cap stocks were excluded, the size premium would be greatly diminished, or disappear altogether. For example, Siegel examined the 9-year period 1975 through 1983, and calculated that over this period "…small stocks averaged a 35.3 percent compound annual rate of return, more than double the 15.7 percent return on large stocks".[62] The study concluded that if this 9-year period is excluded, the size premium (as measured over the time period 1926–2001) is greatly reduced or non-existent.

We do not dispute that over the periods measured and using the stock series employed in the study that this author's conclusions are probably correct. However, it may make little sense to exclude a particular 9-year period from the calculation of a historical average merely because its average premium was greater than that of any other 9-year period.

First, the returns of nearly any security can generally be collapsed by simply excluding the best periods. For example, in Graph 8, $1 invested in Apple Inc. (AAPL, NASDAQ) in December 1992 would have turned into $37.72 by December 2012. However, if the best performing 14 months over the 20-year period are excluded (out of 240 months total), a $1 investment in Apple in December 1992 would have turned into $0.95 by the December 2012, representing a 5 percent *loss* over the 20 year period.[63]

Graph 8: Terminal Index Values as of December 31, 2012 for Apple (AAPL: NASDAQ)
The result of excluding the best 14 months' returns from the previous 240 months (20 years)
Index (Year-end 1992 = $1)
January 1993–December 2012

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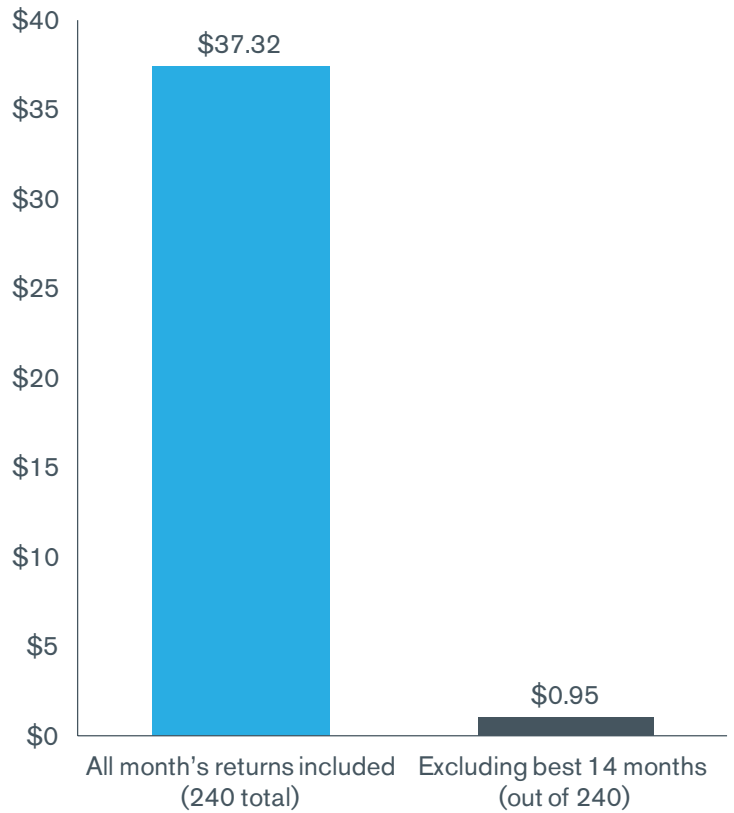
[62] Siegel, Jeremy J., *Stocks for the Long Run*, 3rd edition, McGraw-Hill 2002, pages 134–135. In this study the S&P 500 Index was used to represent large-cap stocks. Small-cap stocks were represented by the bottom quintile (20 percent) size of the NYSE stocks until 1981, and then by the Dimensional fund Advisors (DFA) Small Company Fund from 1982–2000, and then the Russell 2000 Index for 2001.
[63] Calculated by Duff & Phelps. Source of underlying data: Standard & Poor's *Capital IQ* database.

The Size Study

Second, the size effect can vary depending on the indices used or periods examined. For example, in Graph 9 the same 9-year period (1975–1983) is excluded as was excluded in Siegel (1982), and the period examined was extended to December 2012.[64] While the size effect is indeed significantly diminished over the 1926–2001 period, small-cap stocks seem to regain their footing in the following years.

Graph 9: Large-cap Stocks versus Small-cap Stocks
Index (Year-end 1925 = $1)
January 1926–December 2012
January 1975–December 1983 excluded
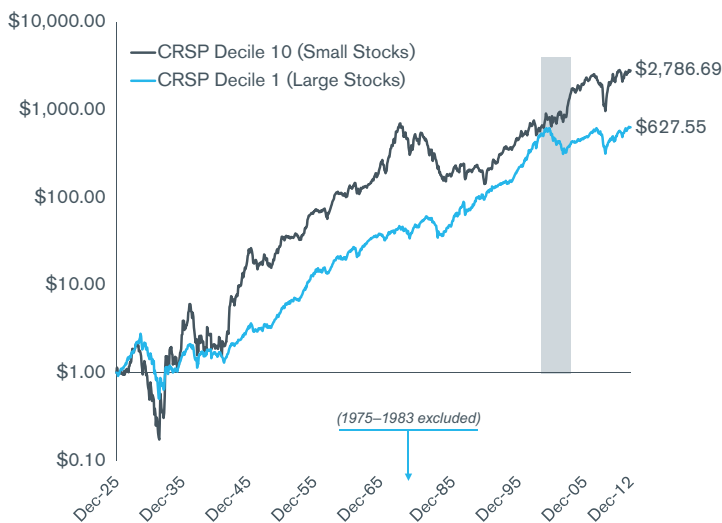
There is little doubt that the period around the turn of the century (the shaded area in Graph 9) was a difficult period for small-cap stocks. The NASDAQ Composite Index, for example, is populated with generally smaller companies than those in the S&P 500 index.[65] While the NASDAQ declined from high of 5,048.62 on March 10, 2000 to 1,950.40 on December 31, 2001 (a loss of approximately 61 percent), the S&P 500 Index declined less than one third as much, from 1,395.07 to 1,148.08 (a loss of approximately 18 percent). Clearly, small-cap company stocks underperformed their larger-cap company stock counterparts by a very significant margin during this period.[66]

Is the Size Effect Limited to Only the Smallest Companies?

Over long periods of time, the size effect is not just evident for the smallest companies, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of several billions of dollars. Summary statistics for CRSP NYSE/AMEX/NASDAQ deciles 1–10 are shown in Table 3.[67] As size as measured by market capitalization decreases, return tends to increase. For example, the average annual arithmetic return of decile 1 (the largest-cap stocks) was 10.88 percent over the 1926–2012 period, while the average annual arithmetic return of decile 10 (the smallest-cap stocks) was 20.56 percent. Note that increased return comes at a price: risk (as measured by standard deviation) increases from 19.09 percent for decile 1 to 44.55 percent for decile 10. The relationship between risk and return is a fundamental principle of finance, and a cost of capital estimate is, in essence, grounded in the relationship between risk and return.[68]

Table 3: Summary Statistics of Annual Returns (CRSP NYSE/AMEX/NASDAQ Deciles)
1926–2012

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Decile 1–Largest	9.10	10.88	19.09
Decile 2	10.44	12.81	22.05
Decile 3	10.84	13.41	23.61
Decile 4	10.79	13.81	25.80
Decile 5	11.34	14.58	26.57
Decile 6	11.31	14.80	27.19
Decile 7	11.27	15.18	29.41
Decile 8	11.48	16.29	33.96
Decile 9	11.49	16.85	36.14
Decile 10–Smallest	13.03	20.56	44.55

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[64] Derived by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. In this example, large-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 1; small-cap stocks represented by CRSP NYSE/AMEX/NASDAQ decile 10. Source: Morningstar *EnCorr software*.

[65] As of December 2012, the average and median market capitalization of S&P 500 index components is approximately $25.5 billion and $12.0 billion, respectively. The average and median market capitalization of companies included in the NASDAQ Composite Index is approximately $1.9 billion and $207 million, respectively. Source: Standard and Poor's *Capital IQ* database.

[66] The equities bull market in the latter half of the 1990's and early 2000's is commonly referred to as the "dot.com bubble" because of the large number of technology companies that arose. During this period, NASDAQ index increased from 1,058.65 on January 2, 1996 to 5,048.62 on March 10, 2000. In the period that followed, the NASDAQ index ultimately declined (i.e., "crashed") to 1,114.11 on October 9, 2002, representing nearly a complete retracement to January 1996 levels.

[67] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.

[68] As paraphrased from: Shannon Pratt and Roger Grabowski, *Cost of Capital: Applications and Examples* 4th ed. (New York; John Wiley & Sons, 2010), page 471.

The Size Study

While there is evidence of the size effect across the size spectrum, the size effect is not "linear" – the effect is greatest in the smallest companies. For example, an investment of $1 in large-cap stocks at the end of 1925 would have grown to $1,951.66 by the end of 2012, while an investment of $1 in small-cap stocks over the same period would have grown to $42,347.63. As illustrated in Graph 10[69], the size effect is clearly concentrated in the smallest-cap companies.[70]

Graph 10: Terminal Index Values of CRSP NYSE/AMEX/NASDAQ Deciles 1–10
Index (Year-end 1925 = $1)
January 1926–December 2012

Graph 11: Terminal Index Values of CRSP NYSE/AMEX/NASDAQ Decile 1 and a Portfolio Comprised of Deciles 6–9
Index (Year-end 1925 = $1)
January 1926–December 2012
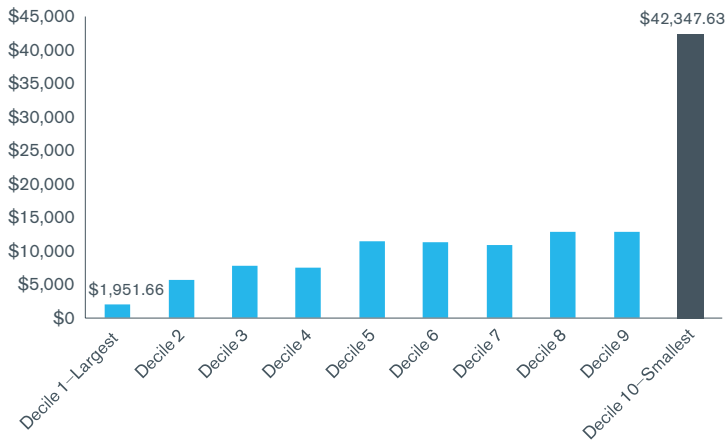
This does not mean, however, that the size effect is present in only the smallest-cap companies. To illustrate this, decile 1 (large-cap stocks) is compared to a portfolio comprised of equal parts of deciles 6–9 in Graph 11.[71] Decile 10, which is comprised of the smallest-cap companies, is *excluded* from the analysis. An investment of $1 in large-cap stocks at the end of 1925 would have grown to $1,951.66 by the end of 2012, while an investment of $1 in the portfolio comprised of deciles 6–9 would have grown to $12,856.83 over the same period. Even with decile 10 excluded, the portfolio made up of deciles 6–9 outperformed large-cap stocks over longer periods.

[69] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.
[70] Some researchers have suggested that the size effect is concentrated in even smaller firms than discussed here. Horowitz, Loughran, and Savin found that if "…firms less than $5 million in value are excluded from the sample universe…", the size effect becomes insignificant, at least as measured over the 1963–1997 time period. Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The disappearing size effect", *Research in Economics* (2000), 83-100.
[71] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.

The Size Study

Has the Size Effect Disappeared in More Recent Periods?

The Risk Premium Report finds that as company size decreases, company risks increase; hence the cost of equity capital for small firms is greater. Some research has suggested that in more recent years the size effect is greatly diminished, or even disappears altogether.

For example, Hou and van Dijk posited that the apparent disappearance of the size effect after the early 1980s was due to cash flow shocks. Realized returns for small companies were generally less than expected because of negative cash flow shocks, and realized returns for large companies were generally greater than expected because of positive cash flow shocks.[72]

What caused the cash flow shocks? The number of newly public firms in the United States increased dramatically in the 1980s and 1990s compared with prior periods, and the profitability and survival rate of the newly public firms was generally less than the profitability and survival rates for firms that went public in previous years. After adjusting realized returns for the cash flow shocks, the result was that returns of small firms on a pro forma basis exceeded the returns of large firms by approximately 10% per annum, consistent with the size premium in prior periods.

A more direct reason often cited for a diminished size effect in more recent years was possibly most succinctly stated by Horowitz, Loughran, and Savin, who suggested that "…it is quite possible that as investors became aware of the size effect, small firm prices increased (thus lowering subsequent returns)."[73] This conjecture may be supported by the sheer number of "small-cap" funds that have come into existence since Banz's 1981 article that demonstrated that smaller-cap stocks exhibited significantly greater performance over the period from 1926 to 1975.[74]

In Graph 12, the annual compound rate of return for large-cap stocks and small-cap stocks was calculated over all 10-year periods from January 1982 to December 2012, and then the simple difference between small-cap stocks' returns and large-cap stocks' returns was then calculated for each period.[75, 76] The first 10-year (120-month) period examined (on the left-hand side of the graph) is the 10-year period from January 1982 to December 1991, and the last 10-year (120-month) period examined (on the right-hand side of the graph) is the 10-year period from January 2003 to December 2012. All of the data used in Graph 12 is thus from periods *after* Banz's article was published in 1981.

The patterns gleaned from examination of Graph 12 are mixed: 10-year periods ending in the 1990s were generally good for large-cap stocks relative to small-cap stocks, and 10-year periods ending in the 2000s were generally good for small-cap stocks relative to large-cap stocks. Overall, small-cap stocks beat large-cap stocks 54% of the time, and large-cap stocks beat small-cap stocks 46% of the time.

Graph 12: Large-cap Stocks (CRSP Decile 1) versus Small-cap Stocks (CRSP Decile 10)
Difference in annual compound rates of return over 10-year holding periods.
January 1982–December 2012

120-month period ending

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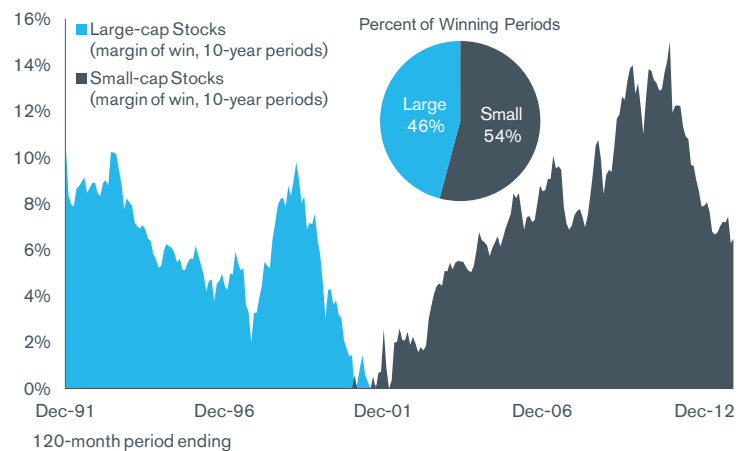
[72] Kewei Hou and Mathijs A. van Dijk, "Resurrecting the size effect: Firm size, profitability shocks, and expected stock returns", Ohio State University Fisher College of Business working paper, July 2012. Copy available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=556203

[73] Joel L. Horowitz, Tim Loughran, and N.E. Savin, "The disappearing size effect", *Research in Economics* (2000), page 98.

[74] Banz, Rolf W. ''The Relationship between Return and Market Value of Common Stocks.'' *Journal of Financial Economics* (March 1981): 3–18. Professor Banz's 1981 article is often cited as the first comprehensive study of the size effect.

[75] Derived by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. In this example, large-cap stocks are represented by CRSP NYSE/AMEX/NASDAQ decile 1; small-cap stocks represented by CRSP NYSE/AMEX/NASDAQ decile 10. Source: Morningstar *EnCorr* software.

[76] There are a total of 253 10-year (i.e., 120-month) periods over the January 1982–December 2012 time horizon.

The Size Study

In the most recent periods, say 2000–2012, small-cap stocks have outperformed large-cap stocks significantly. Referring to Graph 13[77], a $1 investment in December 1999 in CRSP decile 10 (small-cap stocks) would have *increased* to $3.79 by the end of December 2012, while a $1 investment in December 1999 in CRSP decile 1 (large-cap stocks) would have only *increased* to $1.06 by the end of December 2012.

In Table 4[78], summary statistics of CRSP NYSE/AMEX/NASDAQ deciles 1–10 are shown over the time period 2000–2012. The average annual arithmetic return of decile 1 (the largest-cap stocks) was 2.12 percent over the 2000–2012 period (and 0.42 percent measured on geometric basis), while the average annual arithmetic return of decile 10 (the smallest-cap stocks) was 16.62 percent (and 10.78 percent measured on a geometric basis).

Graph 13: Terminal Index Values of CRSP NYSE/AMEX/NASDAQ Deciles 1–10
Index (Year-end 1999 = $1)
January 2000–December 2012

Table 4: Summary Statistics of Annual Returns CRSP NYSE/AMEX/NASDAQ Deciles 1–10
2000–2012

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Decile 1–Largest	0.42	2.12	18.41
Decile 2	5.37	7.78	22.01
Decile 3	5.30	7.79	22.74
Decile 4	5.99	8.38	22.87
Decile 5	6.76	9.03	22.31
Decile 6	6.28	9.06	24.38
Decile 7	6.88	9.68	24.91
Decile 8	8.00	11.16	26.94
Decile 9	7.66	11.17	28.96
Decile 10–Smallest	10.78	16.62	38.50

[77] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.
[78] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.

The Size Study

The Size Effect Tends to Stabilize Over Time

It may be instructive to examine the tendencies of small-cap stocks' performance versus large-cap stocks' performance over time periods with fixed starting dates and variable ending dates. This will help to see what happens as more time periods are added (and thus the importance of "unusual" time periods is diminished).

In Graph 14, the average difference in annual returns for small-cap stocks minus large-cap stocks was calculated for periods with fixed starting dates of 1926 (the first year data is available from CRSP), 1963 (the risk premia and size premia in the *2013 Risk Premium Report* are calculated over the time period 1963–2012), and 1982 (the year following publication of Banz's 1981 article).[79, 80]

On the far left side of Graph 14 for the series "Fixed Beginning Date 1926", the first data point is the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1926–1927, the second data point (moving to the right) is the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1926–1928, and then 1926–1929, etc., until the final data point on the far right is the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1926–2012.

The same analysis is displayed for "Fixed Beginning Date 1963", with the first data point being the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1963–1964, 1963–1965, etc., until the final data point on the far right is the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1963–2012.

And finally, the same analysis for "Fixed Beginning Date 1982" is shown, with the leftmost data point being the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1982–1983, and the rightmost data point being the average difference in annual return for small-cap stocks minus large-cap stocks over the period 1982–2012.

Graph 14 suggests that while the size effect measured over shorter time periods may be quite erratic (and even negative at times), there seems to be an overall tendency toward stability as time periods are added and the longer the period over which it is measured (regardless of the start date). Further, the stability seems to be reached in "positive territory" (the rightmost points in Graph 14), suggesting a positive size effect over time.

Graph 14: CRSP Decile 10 minus Decile 1, Average Difference in Annual Returns
Fixed beginning date, variable ending dates
1926–2012, 1963–2012, 1982–2012

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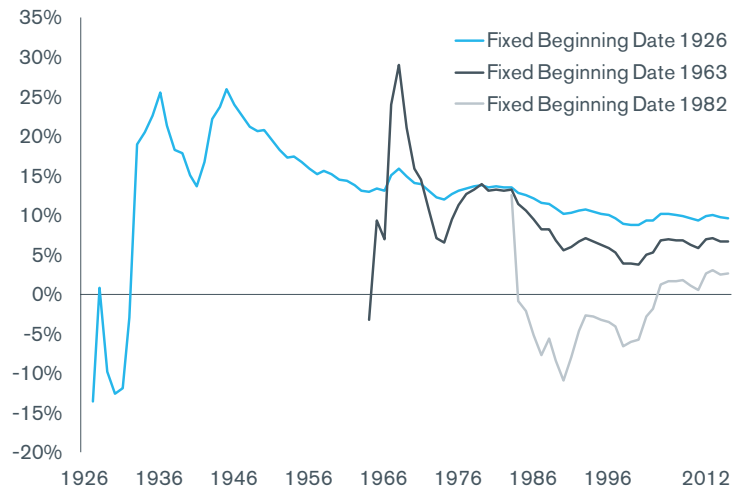
[79] Calculated by Duff & Phelps based on CRSP® data, © 2013 Center for Research in Security Prices (CRSP), University of Chicago Booth School of Business. Source: Morningstar *EnCorr* software.

[80] Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics* (March 1981): 3–18. Banz's 1981 article demonstrated that smaller-cap stocks exhibited significantly greater performance over larger-cap stocks over the period from 1926 to 1975.

The Size Study

The Size Effect and Alternative Measures of Size

In addition to presenting risk premia and size premia for 25 size-ranked portfolios using the traditional "market capitalization" measure, the *Duff & Phelps Risk Premium Report* also considers 7 additional measures of company size, including book value of equity, 5-year average net income, market value of invested capital (MVIC), total assets, 5-year average EBITDA, sales, and number of employees.

The inverse relationship between size and historical rates of return, regardless of how size is measured, is demonstrated in Graph 4, (see page 25). It is clear that over the period that the *2013 Report* is calculated (1963–2012), the average annual return of portfolios sorted by each of the eight size measures tends to increase as "size" decreases. Evidence suggests that a size effect also exists over more recent time periods for the eight size measures examined in the *Risk Premium Report*.

This concept is illustrated in Graphs 15(a) through 15(f) on the following page. As previously discussed, beta does not explain all of the return of smaller companies, and the size premium represents the difference in historical excess returns (i.e. what "actually happened"), and the excess returns predicted by CAPM.[81]

In Graphs 15(a) through 15(f), the security market line (SML) represents what a basic CAPM (i.e., a CAPM with no size adjustment) would predict.[82] In the three graphs on the *left hand side*, 15(a), 15(b), and 15(c), a scatter-plot of the average annual return and betas of the 25 portfolios sorted by *market capitalization* overlay the SML over the time horizons 1963–2012, 1980–2012, and 1990–2012.[83, 84]

In the three graphs on the *right hand side*, 15(d), 15(e), and 15(f), a scatter-plot of the average annual return and betas of the 25 portfolios sorted by *5-year average net income* overlay the SML, also over the time horizons 1963–2012, 1980–2012, and 1990–2012.[85, 86]

For the given level of risk (as implied by beta), one would expect each of the data points to fall neatly upon the SML – this is where CAPM says they *should* be – but they do not. The portfolios' *actual* average returns tend to lie above the SML. The distance above the SML (i.e., the difference between what "actually happened" and what CAPM predicted) is the assumed size premium.

Graphs 15(a) through Graph 15(f) suggest a size effect size over both the longest time horizon examined (1963–2012, which is the period over which the *2013 Report* is calculated), and the shorter time horizons examined (1980–2012 and 1990–2012). Also note that Portfolio 25, which is comprised of the smallest companies, is furthest above the SML in each of the graphs, implying the largest size premium for that portfolio. This is consistent with an inverse relationship between return and size (i.e., as size decreases, return tends to increase). For example, the average smoothed size premium for all eight size measures in the *2013 Report* ranges from an average of 0.42 percent for Portfolio 1 (comprised of the largest companies) to 6.73 percent for Portfolio 25 (comprised of the smallest companies).

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[81] For a detailed discussion of size premia, see *"Risk Premium Over CAPM ("Size Premium"), RP$_s$,"* on page 44.

[82] The SML intersects the risk-free rate on the left axis and a single point scatterplot of the average annual return of the market benchmark (in this case the S&P 500 Index), and the beta of the market benchmark (1.0).

[83] The 25 portfolios sorted by market capitalization are used to calculate risk premia over the risk-free rate in Exhibit A-1, and are used to calculate risk premia over CAPM (i.e, Size Premium) in Exhibit B-1.

[84] In the *2013 Risk Premium Report*, the risk premia and size premia presented in the Data Exhibits are calculated over the time horizon 1963–2012. The custom time horizons shown in Graph 17(a) through 17(f) (1980–2012, 1990–2012) were developed using the same database as was used to create the *2013 Report's* Data Exhibits.

[85] The 25 portfolios sorted by 5-year average net income are used to calculate risk premia over the risk-free rate in Exhibit A-3, and are used to calculate risk premia over CAPM in Exhibit B-3.

[86] While Graphs 15(a) though 15(f) present information for "market capitalization" and "5-year average income" only, the same analysis was performed on the other six size measures analyzed in the *Risk Premium Report*. All eight of the size measures, over 1963–2012, 1980–2012, and 1990–2012 yielded similar results as shown in Graph 15(a) through 15(f).
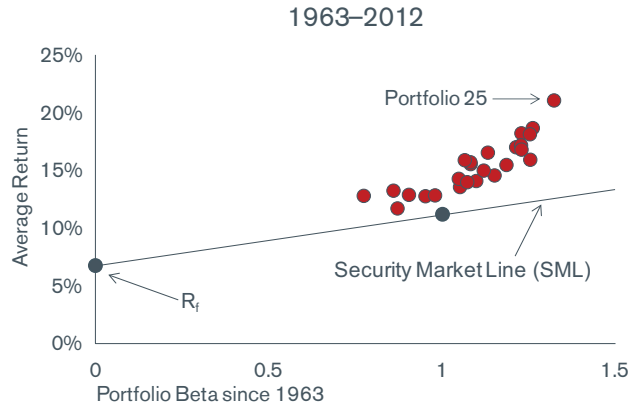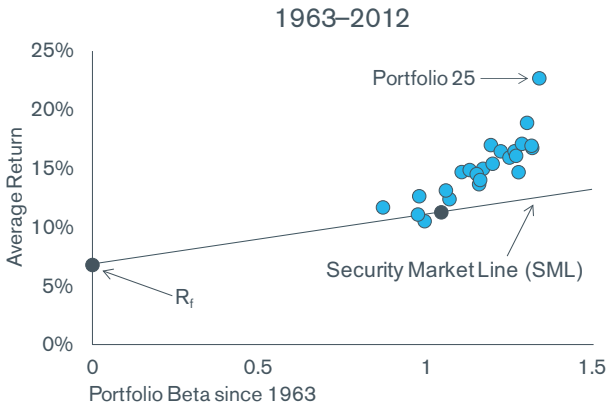
The Size Study

Graphs 15(a), 15(b), 15(c), 15(d), 15(e), 15(f): Security Market Line (SML) versus Size Study Portfolios 1–25
Exhibits B-1 (Market Capitalization) and B-3 (5-Year Avg. Net Income)
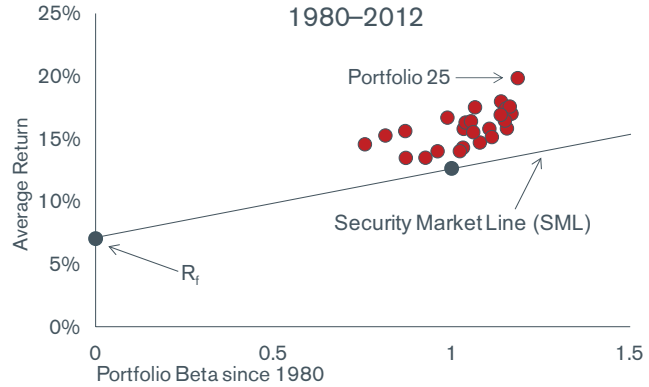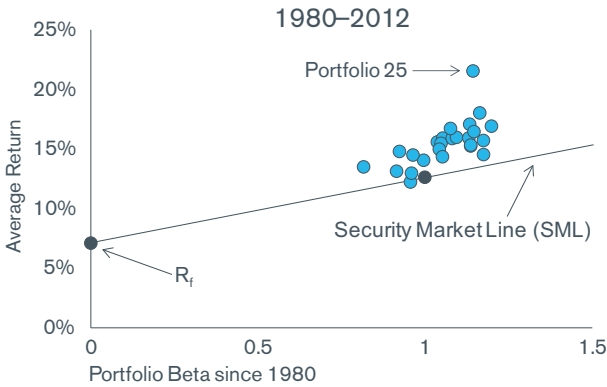1963–2012, 1980–2012, 1990–2012

Market Capitalization

Graph 15(a)

Graph 15(b)

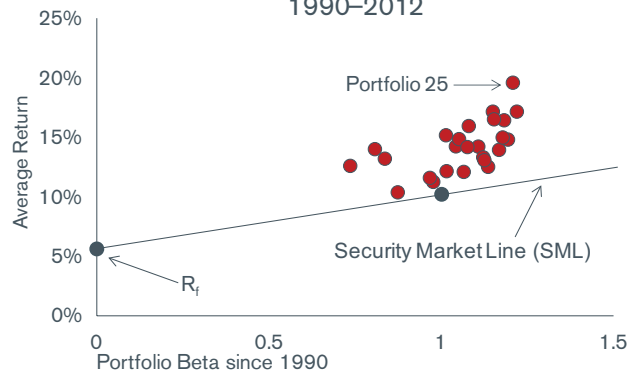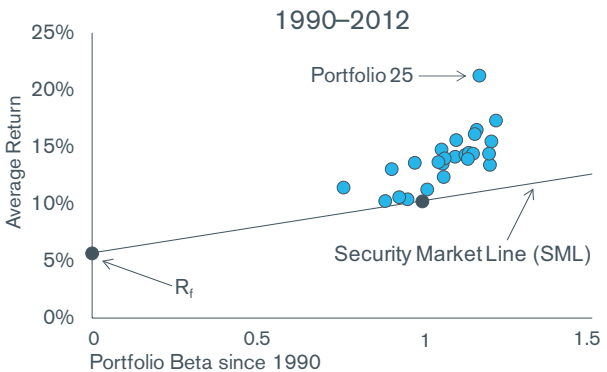### Graph 15(c)

5-Year Avg. Net Income

Graph 15(d)

Graph 15(e)

Graph 15(f)

The Size Study

The January Effect

The "January effect" is the empirical observation that rates of return for small-cap stocks have on the average tended to be greater in January than in the other months of the year. The existence of a January effect, however, does not necessarily present a challenge to the size effect unless it can be established that the effect is the result of a bias in the measurement of returns.

Some academics have speculated that the January effect may be due to a bias related to tax-loss selling. For example, investors who have experienced a loss on a security may be motivated to sell their shares shortly before the end of December. An investor makes such a sale in order to realize the loss for income tax purposes. This tendency creates a preponderance of sell orders for such shares at year-end. If this is true, then (1) there may be some temporary downward pressure on prices of these stocks, and (2) the year-end closing prices are likely to be at the bid rather than at the ask price. The prices of these stocks will then appear to recover in January when trading returns to a more balanced mix of buy and sell orders (i.e., more trading at the ask price).

Such "loser" stocks will have temporarily depressed stock prices. This creates the tendency for such companies to be pushed down in the rankings when size is measured by market value. At the same time, "winner" stocks may be pushed up in the rankings when size is measured by market value. Thus, portfolios composed of small-cap companies tend to have more losers in December, with the returns in January distorted by the tax-loss selling. A recent study finds that the January returns are smaller after 1963–1979 but have reverted to levels that appear before that period.[87] More important, they find that trading volume for small-cap companies in January does not differ from other months. They conclude that the January effect continues.

This argument vanishes if you use a measure other than market value (e.g., net income, total assets, or sales) to measure size because a company's fundamental size does not change in December because of tax loss selling. The size effect is evident in the *Size Study* using size measures other than market capitalization.

Is the Size Effect a Proxy for "Liquidity"?

Banz's 1981 musing as to whether *"…size per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size"* may have been cannily prescient. Research on returns as related to "size" is abundant, but over time a growing body of work investigating the impact of "liquidity" on returns has emerged. As early as 1986, Amihud and Mendelson, demonstrated that "…market-observed average returns are an increasing function of the spread…" (i.e., less liquid stocks, as measured by a larger bid-ask spread, outperform more liquid stocks), and further concluded that the "…higher yields required on higher-spread stocks give firms an incentive to increase the liquidity of their securities, thus reducing their opportunity cost of capital".[88]

Recent research by Abbot and Pratt suggests that the "…difference between mean returns on size sorted portfolios is considerably smaller than the difference between mean returns on liquidity sorted portfolios", implying that between size and liquidity (as measured by a natural log transformation of stock turnover), "…liquidity may be the dominant factor in asset pricing."[89]

Ibbotson, Chen, Kim, and Hu suggest that while the typical measures of liquidity employed in the literature are each "…highly correlated with company size", they demonstrate that liquidity, as measured by annual stock turnover, "…is an economically significant investment style that is just as strong, but *distinct* from traditional investment styles such as size, value/growth, and momentum" [emphasis added].[90] The authors go on to say that "…there is an incremental return from investing in less liquid stocks even after adjusting for the market, size, value/growth, and momentum factors", and conclude that "…equity liquidity is the missing equity style."

Ibbotson, Chen, and Hu identify two main sources of the greater returns of less liquid stocks. The first is that "investors like liquidity and dislike illiquidity", and "…a premium has to be paid for any characteristic that investors demand, and a discount must be given for any characteristic investors seek to avoid". Thus, "…the investor in less liquid stocks gets lower valuations, effectively buying stocks at a discount."

[87] Kathryn E. Easterday, Pradyot K. Sen, and Jens A. Stephan, "The Persistence of the Small Firm/January Effect: Is It Consistent with Investors' Learning and Arbitrage Efforts?" June 2008. Copy available at http://ssrn.com/abstract=1166149

[88] Amihud, Yakov and Haim Mendelson, 1986, "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics* 17, 223-249.

[89] Ashok Abbott and Shannon Pratt, "Does Liquidity Masquerade as Size", working paper, 2012.

[90] See Roger G. Ibbotson, Zhiwu Chen, Daniel Y.-J. Kim, and Wendy Y. Hu, "Liquidity as an Investment Style", August 2012. SSRN id1817889. Copy available at www.zebracapital.com

The Size Study

The second factor is that high liquidity stocks tend to become less liquid, and less liquid stocks tend to become more liquid ("…liquidity tends to mean revert…"). Thus, "…the investor in less liquid stocks also gets the gain from the increase in liquidity." (i.e. as a less liquid stock becomes more liquid, valuations increase).

The *Risk Premium Report* provides data for the user to estimate the COE of a subject company as if the subject company were a publicly-traded company. If one is using the *Risk Premium Report* to estimate the COE of a small, closely held company, the estimated COE reflects the COE for a comparably sized, publicly traded company with the average liquidity characteristics of such a company.

In estimating any adjustment for lack of marketability appropriate for the closely held subject company, the user should match the characteristics of the subject company to the characteristics of the companies from which the lack of marketability data is drawn.[91] For example, if the subject closely held company is established and profitable, its characteristics likely match those of companies for which returns are reported in Portfolio 25 of Exhibit A-2. If one is estimating the adjustment for lack of marketability, using restricted stock discounts, one needs to apply discount data drawn from purchases of restricted stocks of established, profitable companies, not start-up companies. That way there is a matching of the estimated liquidity inherent in the data for companies comprising Portfolio 25 and companies used in estimating the discount for lack of marketability.

The Size Effect: Closing Thoughts

While the size effect does wax and wane, and may even be negative over significant portions of time, small company stocks' outperformance over large company stocks appears to be a persistent trend over the longer term. The size effect is not "linear" – the effect is greatest in the smallest companies, but there is evidence of the size effect across the size spectrum. The size effect exists for alternative measures of size (in addition to the traditional market capitalization). Using alternative measures of size enables greater flexibility, and at the same time enables the analyst to avoid potential "circularity" issues. The size effect may be a proxy for "liquidity" or other risk factors included in the pricing of publicly traded stocks.

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[91] Such an adjustment is commonly made to the resulting indicated value but can also be made by increasing the COE to account for the additional COE of an illiquid investment in a closely held company.

The Size Study

The "A" and "B" Exhibits – Summary of Data Presented

While the A and B exhibits present different types of risk premia, both the A and B exhibits' 25 portfolios are ranked by the *same* eight alternative measures of size, which are described in Table 5.[92]

Each of the exhibits A-1 through A-8 *and* B-1 through B-8 displays one line of data for each of the 25 size-ranked portfolios. The A and B exhibits include the statistics outlined in Table 6 for each of the size measures outlined in Table 5.

For comparative purposes, the average returns from the *SBBI* series for Large Companies (essentially the S&P 500 Index), Small Companies, and Long-Term Government Bond Income Returns for the period 1963 through the latest year are also reported in each exhibit.[93]

Table 5: Eight Alternative Measures of Size

Exhibits A-1 and B-1
Market value of common equity (common stock price times number of common shares outstanding).

Exhibits A-2 and B-2
Book value of common equity (does not add back the deferred tax balance)

Exhibits A-3 and B-3
5-year average net income for previous five fiscal years (net income before extraordinary items).

Exhibits A-4 and B-4
Market value of invested capital (MVIC) (market value of common equity plus carrying value of preferred stock plus long-term debt (including current portion) and notes payable).

Exhibits A-5 and B-5
Total Assets (as reported on the balance sheet).

Exhibits A-6 and B-6
5-year average earnings before interest, income taxes, depreciation and amortization (EBITDA) for the previous five fiscal years (operating income before depreciation plus non-operating income).

Exhibits A-7 and B-7
Sales (net).

Exhibits A-8 and B-8
Number of employees (number of employees, either at year-end or yearly average, including part-time and seasonal workers and employees of consolidated subsidiaries; excludes contract workers and unconsolidated subsidiaries).

Table 6: Statistics Reported for 25 size-ranked portfolios in the *Size Study*'s A and B Exhibits

Exhibits A-1 through A-8

- Average of the sorting criteria (e.g., average number of employees) for the latest year used in determining the size of the companies (i.e., the size criteria when the latest year's portfolios are formed). For example, the market value in Exhibit A-1 is the market value of equity at the beginning of the latest year. The other size criteria are based on what was known at the beginning of the latest year when the portfolios are formed.

- The number of companies in each portfolio at the beginning of the latest year.

- Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).

- Standard deviation of annual historical equity returns.

- Geometric average historical equity return since 1963.

- Arithmetic average historical equity return since 1963.

- Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 (RP_{m+s}).

- "Smoothed" average historical risk premium: the fitted premium from a regression with the average historical risk premium as dependent variable and the logarithm of the average sorting criteria as independent variable. (We present the coefficients and other statistics from this regression analysis in the top right hand corner of the exhibits) (RP_{m+s})

- Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.

Exhibits B-1 through B-8

- Average of the sorting criteria (e.g., average number of employees) for the latest year used in determining the size of the companies (i.e., the size criteria when the latest year's portfolios are formed). For example, the market value in Exhibit B-1 is the market value of equity at the beginning of the latest year. The other size criteria are based on what was known at the beginning of the latest year when the portfolios are formed.

- Beta estimate calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).

- Arithmetic average historical equity return since 1963.

- Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963.

- Indicated CAPM premium, calculated as the beta of the portfolio multiplied by the average historical market risk premium since 1963 (measured as the difference between *SBBI* Large Stock total returns and *SBBI* income returns on long-term Treasury bonds).

- Premium over CAPM, calculated by subtracting the "Indicated CAPM Premium" from the "Arithmetic Risk Premium" (RP_s).

- "Smoothed" Premium over CAPM: the fitted premium from a regression with the historical "Premium over CAPM" as dependent variable and the logarithm of the average sorting criteria as independent variable (RP_s)

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[92] For a detailed description of the Standard and Poor's *Compustat* data items used in the *Risk Premium Report*, please see Appendix A.
[93] Source: Morningstar *EnCorr* software.

The Size Study

The Difference between the A Exhibits and the B Exhibits

The results of the *Size Study* are presented in Exhibits A-1 through A-8 and Exhibits B-1 through B-8. The main difference between the A and B exhibits is how they are used: the A exhibits are used if you are using a "buildup" method to develop cost of equity capital estimates, and the B exhibits are used if you are using the capital asset pricing model (CAPM) to develop cost of equity capital estimates. This difference in usage is a function of the type of "risk premia" presented in each of the exhibits:

- The A exhibits provide risk premia over the risk-free rate in terms of the combined effect of *market risk* and *size risk* for 25 portfolios ranked by eight alternative measures of size (RP_{m+s}). These premia can be added to a risk-free rate (R_f) to estimate cost of equity capital in a "buildup" model.

- The B exhibits provide risk premia over CAPM ("size premia") in terms of *size risk* for 25 portfolios ranked by eight alternative measures of size (RP_s). These premia are commonly known as "beta-adjusted size premia", or simply "size premia". These premia can be added as a size adjustment to a basic CAPM to estimate cost of equity capital.[94]

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[94] The basic CAPM formula is *COE = Risk-Free Rate + (Beta x ERP)*. A "modified CAPM" refers to the common modification to the CAPM formula that is used to incorporate an adjustment for size: *COE = Risk-Free Rate + (Beta x ERP) + Size Premium*. Please note that the modified CAPM as presented is *after* addition of a size premium and *prior* to the addition of any "company-specific" risk premiums that may be applicable.

The Size Study

The Difference Between "Risk Premia Over the Risk-Free Rate" and "Risk Premia Over CAPM"

The *Size Study* measures the relationship between equity returns and up to eight measures of size, including market capitalization. As size decreases, returns tend to increase.

The *Size Study* develops two primary types of risk premia, those that can be added to a risk-free rate if you are using the buildup method (found in Exhibits A-1 through A-8), and premia over CAPM, which are commonly referred to as "beta adjusted size premia", or simply "size premia" (found in Exhibits B-1 through B-8). Size premia can be added as a size adjustment if you are using the capital asset pricing model (CAPM).

Risk Premium Over Risk-Free Rate, RP$_{m+s}$

"Risk premia over the risk-free rate" represent the difference between the historical (observed) return of equities over the risk-free rate. A long-run average historical risk premium is often used as an indicator of the expected risk premium of a typical equity investor. Returns are based on dividend income plus capital appreciation and represent returns after corporate taxes (but before owner-level taxes).

To estimate historical risk premiums, the average rate of return for each of the 25 size-based portfolios is calculated over the sample period, and then the average income return of long-term Treasury bonds (using *SBBI* data) over the same period is subtracted. The result is a clear negative relationship between size and premium over long-term bond yields (i.e. as size decreases, the return over the risk-free rate increases). This difference is a measure of risk in terms of the combined effect of *market risk* and *size risk*.

In Figure 8, for example, an abbreviated version of Exhibit A-6 is shown. The average annual arithmetic return for Portfolio 25 is 20.31 percent over the time period 1963–2012, and the average annual long-term Treasury income return over this period was 6.75 percent. This implies *actual* excess returns of 13.56 percent (20.31% - 6.75%) for this portfolio.

Because these premia have an embedded measure of market (i.e. "beta") risk, these premia are appropriate for use in "buildup" methods that do not already include a measure of market risk, but are *not* appropriate for use in models (e.g. CAPM) that already have a measure of market risk.

- Risk premia over the risk-free rate (*RP$_{m+s}$*) are presented in Exhibits A-1 through A-8. In the *2013 Report*, these risk premia are calculated over the period 1963 (the year that the *Compustat* database was inaugurated) through December 2012.

Figure 8: Calculating Risk Premia Over the Risk-Free Rate (*RP$_{m+s}$*)

Companies Ranked by 5-Year Average EBITDA
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012
Exhibit A-6

Portfolio Rank by Size	Average EBITDA (in $millions)	Log of Average EBITDA	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	*Smoothed Average Risk Premium*	Average Debt/MVIC
1	19,122	4.28	35	0.79	15.93%	11.41%	12.61%	5.86%	*4.08%*	23.08%
2	5,439	3.74	31	0.84	15.69%	11.10%	12.30%	5.55%	*5.64%*	28.89%
///										
25	16	1.19	356	1.30	28.27%	16.78%	**20.31%**	**13.56%**	*12.93%*	22.32%
Large Stocks (Ibbotson *SBBI* data)						9.80%	11.21%	4.46%		
Small Stocks (Ibbotson *SBBI* data)						13.44%	16.17%	9.42%		
Long-Term Treasury Income (Ibbotson *SBBI* data)						6.73%	**6.75%**			

The Size Study

Risk Premium Over CAPM ("Size Premium"), RP$_s$

"Risk Premia over CAPM" represent the difference between historical (observed) excess return and the excess return predicted by CAPM. Years ago, the "small stock premium" was calculated as the simple difference in small company returns versus large company returns.[95] However, an examination of the betas of large stocks versus small stocks revealed that within the context of the capital asset pricing model (CAPM), beta (a measure of market risk) did not fully explain all of the difference between large company returns and small company returns. The observed (i.e., historical) excess return of portfolios comprised of smaller stocks tended to be greater than the excess return predicted by the CAPM.

What followed from this observation is what is now commonly referred to as the "size premium", which can be thought of as the difference in historical excess returns (i.e. "what actually happened"), and the excess returns that CAPM would have predicted.

For each portfolio in the Data Exhibits, a size premium is calculated using the methodology for doing so as described in the *SBBI Valuation Yearbook*.[96] The formula for this adjustment is:

Size Premium = Portfolio Premium − (Portfolio Beta x Realized Market Premium)

where:

Size premium: the difference in historical excess returns (i.e. what "actually happened"), and the excess returns predicted by CAPM.

Portfolio premium: the actual return over the risk-free interest rate (i.e. "excess return") earned by a given portfolio between 1963 and 2012.

Portfolio beta: the beta estimated relative to the S&P 500 Index using annual returns between 1963 and 2012.

Realized market premium: the average annual excess return of the S&P 500 Index between 1963 and 2012 over the long-term risk-free rate.

This adjustment can be thought of as simply "*what actually happened*" (the portfolio premium) minus "*what CAPM predicted would happen*" (the portfolio beta x the realized market premium).[97]

[95] For example, in early versions of what would evolve into the *SBBI Classic Yearbook* (Morningstar, Chicago 2012) the "small stock premium" was calculated as the simple difference between a "small company stock" series and the Standard and Poor's (S&P) Composite Index (i.e., the S&P 500 Index).
[96] *2012 Ibbotson SBBI Valuation Yearbook* (Chicago, Morningstar, 2012), Chapter 7, "Firm Size and Return", pages 85–107.
[97] The basic CAPM equation is *COE = R$_f$ + (ß x ERP)*, which can be rewritten as *COE − R$_f$ = (ß x ERP)*. COE (i.e. "expected return") minus the risk-free rate (R$_f$) is, by definition, the "expected return over the risk-free rate", and therefore, so is *(ß x ERP)*.

The Size Study

For example, an abbreviated version of Exhibit B-6 is shown in Figure 9. The average annual arithmetic return for Portfolio 25 is 20.31 percent over the time period 1963–2012, and the average annual long-term Treasury income return over this period was 6.75 percent. This implies actual excess returns of 13.56 percent (20.31% – 6.75%) for this portfolio.

Portfolio 25 has a calculated beta[98] of 1.30, and the realized market premium over the 1963–2012 period is 4.46 percent.[99] This implies that predicted excess return according to CAPM is 5.80 percent (1.30 x 4.46%) (difference due to rounding).

The size premium for Portfolio 25 in Exhibit B-6 is therefore 7.75 percent, which is "*what actually happened*" (13.56%) minus "*what CAPM predicted*" (5.80%). This is what is meant when we say that the beta of smaller companies doesn't explain all of their returns. In this simple example, beta fell 7.75% short of explaining what actually happened.

The risk premia over CAPM (i.e. "size premia") published in the *Risk Premium Report* are adjusted for beta. In other words, the portion of excess return that is not attributable to beta is controlled for, or removed, leaving only the size effect's contribution to excess return. These premia are appropriate for use in the capital asset pricing model (CAPM), and in buildup methods that do not otherwise already have a measure of size risk.[100]

- Risk premia over CAPM, or "size premia" (RP_s) are presented in Exhibits B-1 through B-8. In the *2013 Report*, these risk premia are calculated over the period 1963 (the year that the *Compustat* database was inaugurated) through December 2012.

Figure 9: Calculating Size Premia (RP_s)

Companies Ranked by 5-Year Average EBITDA
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012
Exhibit B-6

Portfolio Rank by Size	Average EBITDA (in $millions)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	*Smoothed Premium over CAPM*
1	19,122	4.28	0.79	12.61%	5.86%	3.50%	2.35%	*0.65%*
2	5,439	3.74	0.84	12.30%	5.55%	3.73%	1.82%	*1.72%*
⟵				///				⟶
25	16	1.19	**1.30**	**20.31%**	**13.56%**	**5.80%**	**7.75%**	*6.74%*
Large Stocks (Ibbotson *SBBI* data)				11.21%	**4.46%**			
Small Stocks (Ibbotson *SBBI* data)				16.17%	9.42%			
Long-Term Treasury Income (Ibbotson *SBBI* data)				**6.75%**				

[98] The betas presented in the *Risk Premium Report* are "sum" betas. Smaller companies generally trade more infrequently and exhibit more of a "lagged" price reaction (relative to the market) than do large stocks. One of the ways of capturing this lag movement is called sum beta. See Ibbotson, Roger G., Paul D. Kaplan, and James D. Pearson. "Estimates of Small-Stock Betas Are Much Too Low," *Journal of Portfolio Management*, Summer 1997.

[99] As derived from the average difference in the annual average returns of the S&P 500 Index and *SBBI* long-term government Treasury bond income returns. Source: Morningstar *EnCorr* software.

[100] For example, the size premia presented in Exhibit B cannot be used in "Buildup 1". The Buildup 1 method uses "risk premia over the risk-free rate" (from Exhibit A) that already have a measure of risk in terms of the total effect of market risk and size risk, (RP_{m+s}). Using size premia in Buildup 1 would be "double counting" size risk.

The Size Study

Overview of Methods Used to Estimate Cost of Equity Capital Using the Size Study

The *Size Study* provides two methods of estimating COE for a subject company, Buildup 1 and CAPM, plus one method for estimating unlevered COE (the cost of equity capital assuming a firm is financed 100% with equity and 0% debt).[101]

Some users of the *Report* have inquired whether the *Size Study* can be used in conjunction with the industry risk premia (IRPs) published in the *SBBI Valuation Edition Yearbook*, so we also include an alternative method in which a rudimentary adjustment is made to an IRP and then utilized in a modified buildup model, Buildup 2, that includes a separate variable for the industry risk premium.[102] These methods are summarized to the right in equation format, and summarized in Figure 10 in graphical "building blocks" format.

1) **Buildup 1**

$COE_{Buildup\ 1}$ = (Risk-Free Rate) + (Risk Premium in Excess of the Risk-Free Rate) + (Equity Risk Premium Adjustment)

Example 1a: using guideline portfolios: page 51
Example 1b: using regression equations: page 53

2) **Buildup 1-Unlevered**

$COE_{Buildup\ 1\text{-}Unlevered}$ = (Risk-Free Rate) + (Unlevered Risk Premium in Excess of the Risk-Free Rate) + (Equity Risk Premium Adjustment)

Example 2a: using guideline portfolios: page 60
Example 2b: using regression equations: page 64

3) **Capital asset pricing model (CAPM)**

COE_{CAPM} = (Risk-Free Rate) + (Beta x Equity Risk Premium) + (Size Premium)

Example 3a: using guideline portfolios: page 69
Example 3b: using regression equations: page 72

4) **Buildup 2**

$COE_{Buildup\ 2}$ = (Risk-Free Rate) + (Equity Risk Premium) + (Size Premium) + (Adjusted Industry Risk Premium)

Example 4a: using guideline portfolios: page 77
Example 4b: using regression equations: page 78

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[101] Unlevered risk premia over the risk-free rate are presented in Exhibits C-1 through C-8.
[102] Duff & Phelps does not publish IRPs. A source of IRPs is Morningstar's *Ibbotson SBBI Valuation Yearbook*, Table 3-5.

The Size Study

Figure 10: Four Methods of Estimating Cost of Equity Capital with the *Size Study*[103]

Buildup 1

+ ERP Adjustment *

+ Smoothed Risk Premium Over Risk-Free Rate, RP_{m+s}

Risk-Free Rate, R_f

Cost of Equity

(Use Exhibit A risk premia)

CAPM

+ Smoothed Risk Premium Over CAPM ("Size Premium"), RP_s

+ (Beta x ERP)

Risk-Free Rate, R_f

Basic CAPM

Cost of Equity

(Use Exhibit B size premia)

Buildup 1-Unlevered

+ ERP Adjustment *

+ Smoothed Unlevered Risk Premium Over Risk-Free Rate, $RP_{m+s, unlevered}$

Risk-Free Rate, R_f

Cost of Equity

(Use Exhibit C "unlevered" risk premia)

Buildup 2

+ $IRP_{adjusted}$

+ Smoothed Risk Premium Over CAPM ("Size Premium"), RP_s

+ ERP

Risk-Free Rate, R_f

Cost of Equity

(Use Exhibit B size premia)

*ERP Adjustment: The difference between the historical (1963–2012) equity risk premium (ERP) and a user of the *2013 Report*'s own forward ERP estimate:

ERP Adjustment = User's ERP – Historical ERP (1963–2012)

The ERP Adjustment is made only in the "Buildup 1", "Buildup1-Unlevered", "Buildup 1-High-Financial-Risk", "Buildup 3", and "Buildup 3-Unlevered" methods. Please refer to the individual examples provided for these models for more information.

For a detailed discussion of the ERP Adjustment, see page 12.

[103] The relative size of the "building blocks" in Figure 10 do not necessarily represent the relative size of the various inputs.

The Size Study

As shown in Figure 11, there are up to eight alternative size measures that can be used with any of the four methods of estimating COE provided by the *Size Study*. It is important to note that it would not be unusual for fewer than eight of these measures to be available for any given subject company. For example, market value of equity will probably not be available for a closely-held company, nor will market value of invested capital (in which market value of equity is embedded). In cases where fewer than eight size measures are available, it is generally acceptable to use the size measures that *are* available.

Figure 11: Subject Company Size Characteristics (used in all examples)

Size Measure (in $millions, except for Number of Employees)	Appropriate Exhibit			
	Buildup 1	Buildup 1-Unlevered	CAPM	Buildup 2
Market Value of Equity	$120	A-1	C-1	B-1	B-1
Book Value of Equity	$100	A-2	C-2	B-2	B-2
5-year Average Net Income	$10	A-3	C-3	B-3	B-3
Market Value of Invested Capital	$180	A-4	C-4	B-4	B-4
Total Assets	$300	A-5	C-5	B-5	B-5
5-year Average EBITDA	$30	A-6	C-6	B-6	B-6
Sales	$250	A-7	C-7	B-7	B-7
Number of Employees	200	A-8	C-8	B-8	B-8

Figure 11 also includes the data exhibits in which the appropriate risk premia for each of the size measures can be found. For example, for use in the Buildup 1 method, risk premia over the risk-free rate (RP_{m+s}) for "Total Assets" are found in Exhibit A-5. For use in the CAPM method, the appropriate premia over CAPM (RP_s, or "size premia) for "Total Assets" are found in Exhibit B-5.

As discussed previously, the "C" exhibits provide useful information in the form of accounting-based fundamental risk characteristics of each the 25 size-ranked portfolios used in the A exhibits and B exhibits. This important information can be used to gauge whether an increase or decrease to a risk premium or size premium (and thus, cost of equity capital) is indicated, based upon the "company-specific" differences of the subject company' fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived (see *"The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report"* on page 112)

In addition, the C exhibits also provide *unlevered* versions of the risk premia over the risk-free rate found in the A exhibits. These unlevered premia ($RP_{m+s, unlevered}$) can be used to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt.[104]

In each of the following examples of using the *Size Study* to estimate cost of equity capital, the subject company size measures summarized in Figure 11 will be used (total assets of $300 million, for instance, will be used in all examples). Also, the long-term risk-free rate, ERP, and the ERP Adjustment established in the first example (Example 1a, Buildup 1 using "guideline portfolios") will be used (as appropriate) for all the subsequent examples, mirroring the fact that for any given valuation engagement, the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may be (and probably will be) different than the ones used in the examples.

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[104] The D exhibits also include "unlevered" risk premia, but these are unlevered versions of the corresponding "levered" risk premia found in the *Risk Study*'s D exhibits. The unleverered premia in the C exhibits are unlevered versions of the "levered" risk premia found in the *Size Study*'s A exhibits.

The Size Study

Estimating Cost of Equity Capital Using the "Buildup 1" Method
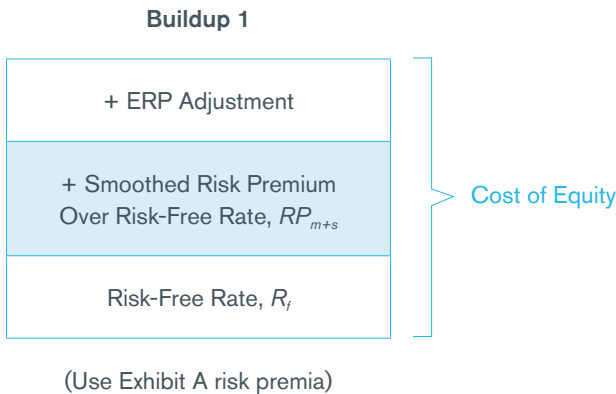
Buildup 1

+ ERP Adjustment
+ Smoothed Risk Premium Over Risk-Free Rate, RP_{m+s}
Risk-Free Rate, R_f

> Cost of Equity

(Use Exhibit A risk premia)

The buildup method is an additive model commonly used for calculating the required rate of return on equity capital. As the name implies, successive "building blocks" are summed, each representing the additional risk inherent to investing in alternative assets. An example of this is the extra return (i.e. "premium"), that investors demand for investing in stocks versus investing in a riskless security.[105]

The *Size Study* calculates average "risk premia over the risk-free rate" from 1963 through December 2012 for each of the 25 size-ranked portfolios by subtracting average annual income returns on long-term U.S. government bonds (using *SBBI* data) from the average annual return of each of the portfolios. The difference is a measure of risk in terms of the combined effect of *market risk* and *size risk* (RP_{m+s}).[106] The result is a clear inverse relationship between size and premium over long-term bond yields. As size decreases, the return over the risk-free rate (i.e. "excess return") tends to increase.

The RP_{m+s} premia are found in Exhibits A-1 through A-8, and can be added to a risk-free rate to estimate cost of equity capital (COE) using the Buildup 1 method.

The Basic Buildup Equation

The basic buildup equation is as follows:

$$E(R_i) = R_f + RP_m + RP_s + RP_u$$

where:

$E(R_i)$ = Expected rate of return on security *i* (this is "cost of equity capital", or "COE")

R_f = Risk-free rate as of the valuation date (typically a long-term US Treasury bond yield)

RP_m = General equity risk premium (i.e. "ERP") estimate for the "market"

RP_s = Risk premium for smaller size (i.e. "premium over CAPM", or "size premium")

RP_u = Risk premium attributable to the specific company or to the industry (the "u" in RP_u stands for <u>u</u>nique risk or company-specific risk, and is also commonly referred to as <u>u</u>nsystematic risk).

In this method, the general equity risk premium for the market is added to the risk-free rate, a premium for small size, and a company-specific risk adjustment, if necessary.[107]

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[105] Throughout this report the risk-free asset is represented by the yield on a 20-year constant maturity Treasury bond.

[106] For a detailed discussion of the risk premia presented in the *Size Study*, see "The Difference Between 'Risk Premia Over the Risk-Free Rate' and 'Risk Premia Over CAPM'" on page 43.

[107] Examples of risks that are typically referred to as "company-specific" risk can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are perceived as unique to the subject company.

The Size Study

The "Buildup 1" Equation

As an alternative to the basic buildup equation, one can use the *Size Study* to develop a risk premium for the subject company for which RP_m (the market premium) and RP_s (the size premium) are combined into a single premium, RP_{m+s}. The basic buildup equation therefore becomes:

$$E(R_i) = R_f + RP_{m+s} + RP_u$$

One final important modification of the basic buildup formula is needed: the "equity risk premium (ERP) Adjustment". The equity risk premium adjustment is made to reconcile the historical data presented in the *Risk Premium Report* with the forward-looking ERP chosen by the individual analyst as of valuation date.[108] For a detailed discussion of the ERP Adjustment, see page 12.

For example, the historical ERP from 1963–2012 (4.5%) was used as an input in performing the analysis in the *2013 Report*, and is thus embedded in the RP_{m+s} premia (the "m" in m+s stands for "market"). However, *Report* users may wish to use a forward-looking ERP estimate as of their valuation dates that differs from the historical 1963–2012 ERP used as a convention to create the *2013 Report*.

For instance, many users of the *2013 Report* use the Duff & Phelps Recommended ERP, which is 5.5 percent as of December 2012.[109, 110] The ERP Adjustment is simply the difference between the user's own forward-looking ERP and the historical 1963–2012 ERP. For example, assuming the user has decided to use 5.5% as his forward-looking ERP, the calculation is:

ERP Adjustment = User's ERP – Historical (1963–2012) ERP, or

1.0% = 5.5% – 4.5%

This implies that on a forward-looking basis as of the valuation date, investors expected to earn 1.0 percent more than they realized on average over the period 1963–2012. It is reasonable to assume that the other historical portfolio returns reported here would differ on a forward-looking basis by a similar differential. Adding the ERP Adjustment to the basic buildup formula produces the full equation for the "Buildup 1" method:

$$COE_{Buildup\ 1} = R_f + RP_{m+s} + RP_u + ERP\ Adjustment$$

The Buildup 1 method is a straightforward way of estimating cost of equity capital (COE) using the historical "risk premiums over the long-term risk-free rate" (RP_{m+s}) presented in Exhibits A-1 through A-8. It is important to understand that because the premia presented in the A exhibits have an embedded measure of market (i.e. "*beta*") risk, they are appropriate only for use in "buildup" methods that do not otherwise include a measure of market risk; these premia are *not* appropriate for use in models (e.g. CAPM) that *already* have a measure of market (beta) risk.[111]

The *Risk Premium Report* provides two ways for analysts to match their subject company's size (or risk) characteristics with the appropriate smoothed premia from the data exhibits: the "guideline portfolio" method and the "regression equation" method.[112] In general, the regression equation method is preferred because this method allows for interpolation between the individual guideline portfolios, although the guideline portfolio method is less complicated, and more direct.

Examples of both the guideline portfolio method and the regression equation method follow, starting with the simpler guideline portfolio method.

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[108] The *ERP Adjustment* is made only in the "Buildup 1", "Buildup1-Unlevered", "Buildup 1-High-Financial-Risk", "Buildup 3", and "Buildup 3-Unlevered" methods.

[109] For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.

[110] See Roger J. Grabowski, "Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of 'Flight to Quality'", *Business Valuation Review, Winter 2010*. A free copy of this article is available at Duff & Phelps' Cost of Capital site at www.DuffandPhelps.com/CostofCapital

[111] Please refer to page 68 for examples illustrating how to use size premia in conjunction with CAPM to estimate COE.

[112] See page 19 for a detailed explanation of the differences between the guideline portfolio method and the regression equation method.

The Size Study

Example 1a: Buildup 1 Method (using guideline portfolios)

Three pieces of information are needed to estimate the cost of equity capital with the Buildup 1 method using "guideline portfolios": a risk-free rate (R_f), a risk premium over the risk-free rate (RP_{m+s}), and an ERP Adjustment. All of the information needed is summarized in Figure 12.

Figure 12: Information Needed to Estimate COE Using Buildup 1 and Guideline Portfolios

Step 1	Step 2	Step 3	Step 4
R_f	RP_{m+s} (using guideline portfolios)	ERP Adj.	COE
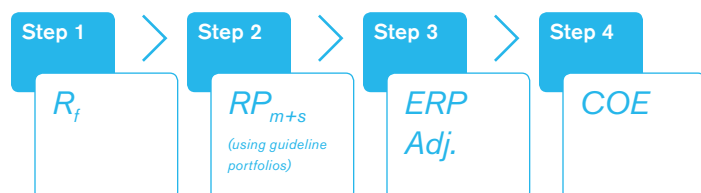
Step 1, Risk-Free Rate (R_f): Because the average annual income return on a long-term (20-year) Treasury bond (6.75% for 1963 through 2012)[113] was used in the calculation of the historical risk premia calculations in Exhibits A-1 through A-8, a 20-year Treasury bond yield is the most appropriate measure of the risk-free rate for use with the *Report*'s risk premiums. For the purposes of this example, a normalized long-term treasury yield of 4.0 percent as of the valuation date is assumed.[114] For a detailed discussion of "normalized" risk-free rates, see page 128.

Step 2, Risk Premium Over Risk-Free Rate (RP_{m+s}): Match the various size measures of the subject company with the guideline portfolios composed of companies of similar size in Exhibits A-1 through A-8, and identify the corresponding smoothed average risk premium.

The subject company in this example has a market value of equity of $120 million, and the appropriate data exhibit is Exhibit A-1 (see Figure 11 on page 48). An abbreviated version of Exhibit A-1 is shown in Figure 13. Of the 25 portfolios, the portfolio that has an average market value *closest* to the subject company's $120 million market value is Portfolio 25 ($94 million). The corresponding smoothed average risk premium is 13.65 percent (13.6 percent, rounded).

Match each of the subject company's size measures in this fashion. For example, the second size measure for the hypothetical subject company in this example is "book value of equity" of $100 million. Of the 25 guideline portfolios in Exhibit A-2 (not shown here), the portfolio that has an average book value of equity *closest* to the subject company's $100 million book value is Portfolio 25 ($67 million). The corresponding smoothed average risk premium is 12.13 percent. After all of the available size measures for the subject company have been matched to the closest guideline portfolio in the appropriate exhibit and the corresponding smoothed average risk premium has been identified for each, Step 2 is complete.

Figure 13: Exhibit A-1 (abbreviated)

Companies Ranked by Market Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Mkt Value (in $millions)	Log of Average Mkt Val.	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	*Smoothed Average Risk Premium*	Average Debt/ MVIC
1	136,859	5.14	39	0.83	16.04%	10.48%	11.72%	4.96%	*2.63%*	14.13%
2	39,247	4.59	30	0.95	16.73%	9.18%	10.54%	3.79%	*4.52%*	19.26%
3	25,711	4.41	34	0.93	15.77%	9.86%	11.11%	4.36%	*5.16%*	20.82%
					///					
24	288	2.46	105	1.25	24.97%	16.16%	18.91%	12.16%	*11.95%*	25.22%
25	94	1.97	273	1.28	30.02%	18.85%	22.70%	15.95%	*13.65%*	27.99%

[113] Source: Morningstar *EnCorr* software.

[114] 2.5 percent is the 20-year constant maturity Treasury bond yield as of December 31, 2012. Refer to Table 13 on page 129 for monthly nominal (i.e., "spot") and "normalized" risk-free rates from 2008 to present. Source of spot rates: The Board of Governors of the Federal Reserve System. Please visit www.federalreserve.gov/datadownload/ for more information. Note that throughout the *Report* the risk-free asset is represented by the yield on a 20-year constant maturity Treasury bond.

The Size Study

Step 3, Equity Risk Premium (ERP) Adjustment: In performing the analysis necessary to produce the *2013 Report*, the historical equity risk premium (ERP) calculated over the time horizon 1963–2012 was used as an input (4.5%).[115] The ERP Adjustment is needed to account for any difference in the user's own ERP estimate and the historical (1963–2012) ERP used to calculate (and thus embedded) in the RP_{m+s} premium. The ERP Adjustment is calculated in the following manner:

ERP Adjustment = User's ERP – Historical (1963–2012) ERP

For the purposes of this example, a forward-looking ERP of 5.5 percent as of the valuation date is assumed, and the ERP Adjustment is calculated as:

1.0% = (5.5% – 4.5%).

This implies that on a forward-looking basis as of the valuation date, investors expected to earn 1.0 percent more than they realized on average over the period 1963–2012, and Buildup 1 COE estimates are adjusted by adding this amount.

Step 4, Estimate Cost of Equity (COE): With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using the Buildup 1 method (using guideline portfolios) is now complete. The risk premia over the risk-free rate (RP_{m+s}) can be added to the risk-free rate (R_f) and the ERP Adjustment to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 14.

The range of COE estimates for the hypothetical subject company in this example is 16.1 percent to 18.7 percent, with an average of 17.4 percent, and a median of 17.2 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" estimate in the group.

Remember that the full Buildup 1 equation is:

$COE_{Buildup\ 1} = R_f + RP_{m+s} + RP_u + ERP\ Adjustment$

The base COE estimates derived in this example are therefore *prior* to the addition of any company-specific risk premiums (RP_u) that the individual analyst may deem appropriate.

Figure 14: Buildup 1 COE Inputs (using guideline portfolios)

	Size Measure (in $millions)	Appropriate Exhibit	Guideline Portfolio	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+s}		Step 3 ERP Adjustment		Step 4 COE*
Market Value of Equity	$120	A-1	25	4.0%	+	13.6%	+	1.0%	=	**18.7%**
Book Value of Equity	$100	A-2	25	4.0%	+	░░░░	+	1.0%	=	░░░░
5-year Average Net Income	$10	A-3	24	4.0%	+	░░░░	+	1.0%	=	░░░░
Market Value of Invested Capital	$180	A-4	25	4.0%	+	░░░░	+	1.0%	=	░░░░
Total Assets	$300	A-5	24	4.0%	+	░░░░	+	1.0%	=	░░░░
5-year Average EBITDA	$30	A-6	25	4.0%	+	░░░░	+	1.0%	=	░░░░
Sales	$250	A-7	24	4.0%	+	11.0%	+	1.0%	=	**16.1%**
Number of Employees	200	A-8	25	4.0%	+	░░░░	+	1.0%	=	░░░░
		Mean (average) values		4.0%	+	12.3%	+	1.0%	=	**17.4%**
		Median (typical) values		4.0%	+	12.1%	+	1.0%	=	**17.2%**

Note: Some values intentionally blurred.

* Difference(s) due to rounding.

[115] Calculated as the annual *SBBI* Large Company Stock (essentially the S&P 500 Index) return minus the average annual long-term *SBBI* government bond income return over the time horizon 1963–2012. Source: Morningstar *EnCorr* software.

The Size Study

Example 1b: Buildup 1 Method (using regression equations)

When the subject company size measures do not exactly match the respective average company size of the guideline portfolios, the data exhibits provide a straightforward way to interpolate an "exact" risk premium over the risk-free rate between guideline portfolios using the "regression equation" method.

The *only* difference between estimating cost of equity capital using the Buildup 1 method using "guideline portfolios" (as in the previous example) and cost of equity capital using the Buildup 1 method using "regression equations" is how the risk premia over the risk-free rate (RP_{m+s}) are identified in Step 2.

Figure 15: Information Needed to Estimate COE Using Buildup 1 and Regression Equations

In the previous example, the smoothed average risk premia *published* in the *Report* for the appropriate guideline portfolios were used to estimate cost of equity capital.[116] In this example, however, the regression equations found in each of the data exhibits will be used to calculate "custom" interpolated risk premia, based upon the specific size measures of the subject company.

This example utilizes the long-term risk-free rate (R_f) and the ERP Adjustment established in a previous example (the *Size Study*'s Buildup 1 method using "guideline portfolios"; see page 51). This mirrors the fact that for any given valuation engagement the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples. The only missing ingredients needed to estimate cost of equity capital are the risk premia over the risk-free rate (RP_{m+s}), as summarized in Figure 16.

Figure 16: Needed—Smoothed Premia (RP_{m+s}) Calculated Using Regression Equations

	Size Measure (in $millions)	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+s}		Step 3 ERP Adjustment		Step 4 COE
Market Value of Equity	$120	A-1	4.0%	+	?	+	1.0%	=	
Book Value of Equity	$100	A-2	4.0%	+	?	+	1.0%	=	
5-year Average Net Income	$10	A-3	4.0%	+	?	+	1.0%	=	
Market Value of Invested Capital	$180	A-4	4.0%	+	?	+	1.0%	=	
Total Assets	$300	A-5	4.0%	+	?	+	1.0%	=	
5-year Average EBITDA	$30	A-6	4.0%	+	?	+	1.0%	=	
Sales	$250	A-7	4.0%	+	?	+	1.0%	=	
Number of Employees	200	A-8	4.0%	+	?	+	1.0%	=	
	Mean (average) values		4.0%	+		+	1.0%	=	
	Median (typical) values		4.0%	+		+	1.0%	=	

[116] The smoothed risk premia published in the *Risk Premium Report* are based upon the average size (or fundamental risk) measure of each of the respective guideline portfolios.

The Size Study

Step 1, Risk-Free Rate (R_f): The risk-free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the normalized long-term treasury yield of 4.0 percent established in Example 1a (page 51).

Step 2, Risk Premium Over the Risk-Free Rate (RP_{m+s}): The hypothetical subject company in this example has a market value of equity of $120 million, and the appropriate *Size Study* data exhibit to use is Exhibit A-1.[117] In this case one would expect that the smoothed average risk premium over the risk-free rate would fall somewhere between 11.95 percent (the smoothed risk premium over the risk-free rate for Portfolio 24) and 13.65 percent (the smoothed risk premium over the risk-free rate for Portfolio 25), as illustrated in Figure 17:

An easy way to calculate a custom interpolated risk premium over the risk-free rate (RP_{m+s}) "in between" Portfolio 24 and Portfolio 25 is by using the regression equations provided for this purpose in each of the data exhibits. The regression equations are located in the same spot in each of the *Size Study*'s exhibits (see Figure 7 on page 20).[118]

The regression equation provided in Exhibit A-1, which includes 25 portfolios ranked by market value[119], is:

*Smoothed Risk Premium = 20.520% − 3.483% * Log (Market Value)*

To calculate an interpolated risk premium, substitute the subject company's $120 million market value into the regression equation as follows[120]:

*Smoothed Risk Premium (RP_{m+s}) = 20.520% − 3.483% * Log (120)*

*13.3% = 20.520% − 3.483% * 2.08*

Continue interpolating smoothed risk premium for each of the size measures available for the subject company using the regression equations from the data exhibits. For example, the second size measure for the subject company is "book value of equity" of $100 million. The regression equation provided in Exhibit A-2 is:

*Smoothed Risk Premium = 16.998% − 2.670% * Log (Book Value)*

Figure 17: Exhibit A-1 (abbreviated)

Companies Ranked by Market Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Mkt Value (in $millions)	Log of Average Mkt Value	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	*Smoothed Average Risk Premium*	Average Debt/MVIC
1	136,859	5.14	39	0.83	16.04%	10.48%	11.72%	4.96%	*2.63%*	14.13%
2	39,247	4.59	30	0.95	16.73%	9.18%	10.54%	3.79%	*4.52%*	19.26%
3	25,711	4.41	34	0.93	15.77%	9.86%	11.11%	4.36%	*5.16%*	20.82%
					///					
24	288	2.46	105	1.25	24.97%	16.16%	18.91%	12.16%	*11.95%*	25.22%
Subject Company	*120*								*?*	
25	94	1.97	273	1.28	30.02%	18.85%	22.70%	15.95%	*13.65%*	27.99%

[117] The same eight size measures for a hypothetical subject company are used in all examples of estimating COE using the *Size Study*, as outlined in Figure 11 on page 48.

[118] The *Risk Study*'s D exhibits also provide regression equations for easy interpolation of risk premia between guideline portfolios of the D exhibits, as do the C exhibits (for unlevered risk premia).

[119] Figure 11 on page 48 lists the appropriate A exhibits in which the risk premia for each of the eight size measures can be found.

[120] Please note that the logarithmic relationship is base-10, and that the financial size data is in millions of dollars, such that the log of $10 million is log (10), not log (10,000,000). The formula to calculate a value's base-10 logarithm in Microsoft Excel is "=log (value)". The "*" used in the regression equation is the symbol used in Microsoft Excel to denote the multiplication symbol, "x". The "*" format is also used to denote multiplication in the regression equations in the data exhibits.

The Size Study

The interpolated smoothed risk premium for "book value of equity" is therefore 11.7 percent (16.998% − 2.670% * 2). After interpolating smoothed risk premia for each of the subject company's available size measures, Step 2 is complete (see Figure 18).

Step 3, Equity Risk Premium (ERP) Adjustment: The ERP Adjustment is needed to account for any difference in the user's own ERP estimate and the historical (1963–2012) ERP. This example utilizes the ERP Adjustment (1.0%) established in Example 1a (page 51).

Step 4, Estimate Cost of Equity (COE): With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using Buildup 1 (using regression equations) is now completed.

Figure 18: Step 2, Calculation of Risk Premia Over the Risk-Free Rate (RP_{m+s}) (using regression equations)

Appropriate Exhibit	Size Measure	Subject Company Size Measures (in $millions)	Appropriate Regression Equation		Smoothed Risk Premium Over Risk-Free Rate, RP_{m+s}
A-1	Market Value of Equity	$120	Smoothed Premium = 20.520% − 3.483% * Log(Market Value)	=	*13.3%*
A-2	Book Value of Equity	$100	Smoothed Premium = 14.80% − 14.80% Log(Book Value)	=	14.80%
A-3	5-year Average Net Income	$10	Smoothed Premium = 14.80% − 14.80% Log(Net Income)	=	14.80%
A-4	Market Value of Invested Capital	$180	Smoothed Premium = 14.80% − 14.80% Log(MVIC)	=	14.80%
A-5	Total Assets	$300	Smoothed Premium = 14.80% − 14.80% Log(Assets)	=	14.80%
A-6	5-year Average EBITDA	$30	Smoothed Premium = 14.80% − 14.80% Log(EBITDA)	=	14.80%
A-7	Sales	$250	Smoothed Premium = 14.80% − 14.80% Log(Sales)	=	14.80%
A-8	Number of Employees	200	Smoothed Premium = 14.80% − 14.80% Log(Employees)	=	14.80%

Note: Some values intentionally blurred.

The Size Study

The "custom" smoothed risk premiums over the risk-free rate (RP_{m+s}) from Figure 18 can now be added to the risk-free rate (R_f) and the ERP Adjustment to estimate an indicated cost of equity capital for the subject company, as illustrated in Figure 19.

The range of cost of equity capital estimates for the hypothetical subject company in this example is 16.3 percent to 18.3 percent, with an average of 17.2 percent, and a median of 17.1 percent. The mean estimate is the simple average of the estimates, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median estimate is generally preferred to the mean. The median tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" estimate in the group.

Remember that the full Buildup 1 equation is:

$COE_{Buildup\ 1} = R_f + RP_{m+s} + RP_u + ERP\ Adjustment$

The base COE estimates derived in this example are therefore *prior* to the addition of any company-specific risk premia (RP_u) that the individual analyst may deem appropriate.

Figure 19: Buildup 1 COE Inputs (using regression equations)

	Size Measure (in $millions)	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+s}		Step 3 ERP Adjustment		Step 4 COE*
Market Value of Equity	$120	A-1	4.0%	+	13.3%	+	1.0%	=	18.3%
Book Value of Equity	$100	A-2	4.0%	+	14.83%	+	1.0%	=	14.83%
5-year Average Net Income	$10	A-3	4.0%	+	14.83%	+	1.0%	=	14.83%
Market Value of Invested Capital	$180	A-4	4.0%	+	14.83%	+	1.0%	=	14.83%
Total Assets	$300	A-5	4.0%	+	14.83%	+	1.0%	=	14.83%
5-year Average EBITDA	$30	A-6	4.0%	+	14.83%	+	1.0%	=	14.83%
Sales	$250	A-7	4.0%	+	11.3%	+	1.0%	=	16.3%
Number of Employees	200	A-8	4.0%	+	14.83%	+	1.0%	=	14.83%
		Mean (average) values	4.0%	+	12.2%	+	1.0%	=	17.2%
		Median (typical) values	4.0%	+	12.1%	+	1.0%	=	17.1%

Note: Some values intentionally blurred.

* Difference(s) due to rounding.

The Size Study

Unlevered Cost of Equity Capital

Starting with the *2011 Report*, the methodology and assumptions for unlevering risk premiums reported in Exhibits C-1 through C-8 were updated.[121] Unlevered premia are used to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt. Generally, as the percentage of leverage (debt) in a company's capital structure increases, the cost of equity capital increases.

The unlevered realized risk premiums displayed in Exhibits C-1 through C-8 are also informative in that they generally indicate that the market views smaller companies' operations to be riskier than the operations of larger companies (i.e., unlevered risk premiums increase as size decreases).

Overview of the Current Methodology and Assumptions Used to Unlever Risk Premia in the 2013 Risk Premium Report

The average (levered) risk premia presented in Exhibits A-1 through A-8 are unlevered as follows[122]:

$$RP_{unlevered} = RP_{levered} - [(W_d / W_e) \times (\beta_u - \beta_d) \times RP_m]$$

where:

$RP_{unlevered}$ = Unlevered realized risk premium over the risk-free rate

$RP_{levered}$ = Levered realized risk premium over the risk-free rate

β_u = Unlevered equity beta[123]

β_d = Debt beta, assumed equal to 0.1

RP_m = General equity risk premium (ERP) estimate for the "market", represented by the average historical risk premium since 1963

W_d = Percent of debt capital in capital structure

W_e = Percent of equity capital in capital structure

The average debt to equity (W_d / W_e) ratio of the portfolio is based on the average debt to MVIC for the portfolio since 1963. A debt beta (β_d) of 0.1 is assumed, which is the average estimated debt beta for the companies included in portfolios 1 through 25 over the years 1963 through 2012 after excluding high-financial-risk companies (high-financial-risk companies are excluded from the base set of companies used in the analysis performed in the *Size Study* and analyzed separately in the *High-Financial-Risk Study*).

A debt beta greater than zero indicates debt capital is bearing risk of variability of operating net cash flow in that interest payments and principal repayments may not be made when owed, inferring that tax deductions on the interest expense may not be realized in the period in which the interest is paid.[124] Preferred capital is included with debt capital in measuring the effect of leverage on the risk of equity capital, which is consistent with recent research.[125]

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[121] Also updated were Exhibits C-1 through C-8 for the *2010 Duff & Phelps Risk Premium Report*, applying the same (updated) methodology and assumptions. The updated 2010 Exhibits C-1 through C-8 can downloaded at www.DuffandPhelps.com/CostofCapital

[122] Derived from R.S. Harris and J. J. Pringle, "Risk-Adjusted Discount Rates – Extensions from the Average Risk Case," *Journal of Financial Research* (Fall 1985) 237-244. Also see: *Arzac, Enrique R., and Lawrence R. Glosten.* "A Reconsideration of Tax Shield Valuation." *European Financial Management (2005): 453-461*. For a more complete discussion see chapter eleven in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

[123] Unlevered betas are often called "asset" betas because they represent the risk of the operations of the business with the risk of financial leverage removed.

[124] For a more complete discussion see Chapter 11 in *Cost of Capital: Applications and Examples* 4th ed. by Shannon Pratt and Roger Grabowski, Wiley (2010).

[125] C.S. Agnes Cheng, C.Z. Liu, K. Newberry, and K.J. Reichelt, "Should Preferred Stock be Classified as a Liability? Evidence from Implied Cost of Equity Capital," working paper (September 2007). Copy available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1014259

The Size Study

An example of unlevering the average risk premia from the A exhibits is demonstrated using the information found in Figure 20a, 20b, and 20c (these are abbreviated versions of Exhibits A-2, B-2, and C-2, respectively).

The average unlevered risk premium of Portfolio 25 in Exhibit C-2 (Figure 20c) is 10.50 percent, calculated using the following information from Figure 20a, Figure 20b, and Figure 20c:

- The arithmetic average risk premium of Portfolio 25 in Exhibit A-2 (see Figure 20a) is 11.72 percent.

- The debt to market value of equity (W_d / W_e) of Portfolio 25 in Exhibit C-2 (see Figure 20c) is 31.01 percent.

- The unlevered sum beta (β_u) of Portfolio 25 in Exhibit C-2 (see Figure 20c) is 0.98.

- The debt beta (β_d) is an assumed 0.1, as discussed previously.

- The market premium (RP_m) used to perform the analysis in the *2013 Report* is the historical ERP from 1963–2012, 4.5%.[126]

To unlever the average (levered) risk premium in Exhibit A-2 (11.72%), substitute these values into the unlevering equation presented earlier:

$$RP_{unlevered} = RP_{levered} - [(W_d / W_e) \; x \; (\beta_u - \beta_d) \; x \; RP_m]$$

$$10.50\%* = 11.72\% - [(31.01\% \; x \; (0.98 - 0.1) \; x \; 4.5\%)]$$

Figure 20a: Exhibit A-2 (abbreviated)

Companies Ranked by Book Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Book Val. (in $millions)	Beta (SumBeta) Since '63	Arithmetic Average Risk Premium
1	50,577	0.81	5.20%
2	15,738	0.86	5.22%
←	///		→
25	67	1.26	11.72%

Figure 20b: Exhibit B-2 (abbreviated)

Companies Ranked by Book Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Book Val. (in $millions)	Premium over CAPM
1	50,577	1.59%
2	15,738	1.41%
←	///	→
25	67	6.11%

Figure 20c: Exhibit C-2 (abbreviated)

Companies Ranked by Book Value of Equity: Comparative Risk Characteristics
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Book Val. (in $millions)	Average Debt to Market Value of Equity	Average Unlevered Risk Premium	Average Unlevered Beta
1	50,577	30.89%	4.45%	0.64
2	15,738	38.88%	4.28%	0.64
←	///			→
25	67	31.01%	10.50%	0.98

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* Difference due to rounding.

[126] Derived as the average annual difference between *SBBI* Large Stock total returns (essentially the S&P 500 index) and *SBBI* income returns on long-term Treasury bonds over the time period 1963–2012.
Source: Morningstar *EnCorr* software.

The Size Study

Unlevered Risk Premia – Reconciliation of the A, B and C Exhibits

Reconciliation of the levered and unlevered betas for use in CAPM (found in Exhibits B-2 and C-2, respectively) now reconcile with the levered and unlevered arithmetic average risk premia for the buildup (found in Exhibits A-2 and C-2, respectively), as demonstrated below using the values from the previous example:

Levered risk premium = Levered beta x Historical market risk premium + Premium over CAPM (i.e. "size premium")

11.7%* = 1.26 x 4.5% + 6.11%

Unlevered risk premium = Unlevered beta x Historical market risk premium + Premium over CAPM (i.e. "size premium")

10.5%* = 0.98 x 4.5% + 6.11%

Relevering

What if the debt-to-market-value-of-equity ratio (W_d/W_e) of the subject company is different than the average (W_d/W_e) of the companies making up Portfolio 25 (31.01% in this case)? It may be possible to adjust the (levered) risk premiums over the risk-free rate (RP_{m+s}) from Exhibits A-1 through A-8 for differences in financial leverage between the subject company and the given guideline portfolio.[127] Again, the average (levered) risk premia presented in Exhibits A-1 through A-8 are unlevered as follows:

$$RP_{unlevered} = RP_{levered} - [(W_d/W_e) \times (\beta_u - \beta_d) \times RP_m]$$

The unlevered risk premia in the C exhibits, which assume a firm is financed 100% with equity and 0% debt, are calculated by unlevering the average risk premia in the A exhibits. In the example, the unlevered risk premium over the risk-free rate ($RP_{m+s, unlevered}$) for Portfolio 25 in Exhibit C-2 (10.50%) was calculated by unlevering the average risk premium over the risk-free rate (RP_{m+s}) for Portfolio 25 in Exhibit A-2 (11.72%). This calculation was performed assuming the 31.01 percent average debt-to-market-value-of-equity ratio (W_d/W_e) of the companies making up Portfolio 25.[128] The percentage of debt in the capital structure went from 31.01 percent to 0 percent, and the unlevered risk premia is lower than the levered risk premium.

This formula can be rearranged to "relever":

$$RP_{levered} = RP_{unlevered} + [(W_d/W_e) \times (\beta_u - \beta_d) \times RP_m]$$

If the subject company has a W_d/W_e ratio that is less (say 20%) than the average W_d/W_e of the guideline portfolio (31.01%), the unlevered risk premium may be "relevered" at the subject company's lower ratio:

11.3% = 10.50% + [(20%) x (0.98 − 0.1) x 4.5%]

The subject company has less debt relative to equity than the average company in the guideline portfolio (20% versus 31.01%), and the relevered risk premium is lower than the average levered risk premium of the guideline portfolio (11.3% versus 11.7%). Generally, as the percentage of leverage (debt) in a company's capital structure decreases, risk to equity investors decreases (and vice versa).
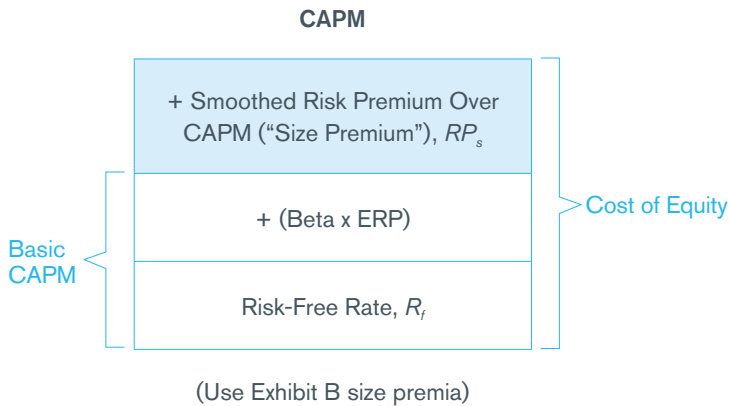
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*   Difference due to rounding.

[127] If one "relevers" at a debt to equity (W_d/W_e) ratio different than the average of W_d/W_e of the given portfolio, other assumptions may not hold. For example, a debt beta of 0.1 is assumed in the unlevering calculations performed in the *Report*. If one relevers at a W_d/W_e ratio that is significantly higher than the average W_d/W_e ratio of the given guideline portfolio, a higher debt beta than 0.1 may be expected, all things held the same.

[128] As found in Exhibit C-2. It is important to understand that each of the A, B, and C exhibits is sorted by *different* size criteria. For instance, the base set of companies used to perform the analyses in the *Size Study* is sorted by "book value of equity", and then used to calculate the different data and information presented in the A-2, B-2, and C-2 exhibits. Citing the present unlevering/relevering example, the average debt-to-market-value-of-equity ratio (W_d/W_e) of the smallest companies (Portfolio 25) *as sorted by* "*book value of equity*" is found in Exhibit C-2, while the average debt-to-market-value-of-equity ratio (W_d/W_e) of the smallest companies (Portfolio 25) *as sorted by* "*total assets*" is found in Exhibit C-5.

The Size Study

Estimating Cost of Equity Capital Using the "CAPM" Method

CAPM

+ Smoothed Risk Premium Over CAPM ("Size Premium"), RP_s
+ (Beta x ERP)
Risk-Free Rate, R_f

Basic CAPM · Cost of Equity

(Use Exhibit B size premia)

The capital asset pricing model (CAPM) is the most widely used method for estimating the cost of equity capital. For example, one survey found that while many firms use multiple methods of estimating the cost of equity capital, 75% of them use the CAPM.[136] Despite its criticisms, the CAPM has been one of the most widely used models for estimating the cost of equity capital for more than 30 years. The basic CAPM formula for estimating the cost of equity capital (COE) is:

$$COE_{CAPM} = R_f + (ß \ x \ ERP)$$

where:

R_f = the risk-free rate as of the valuation date (typically a long-term US Treasury bond yield)

$ß$ = a measure of market (called systematic) risk of a stock; the sensitivity of changes in the returns (dividends plus price changes) of a stock relative to changes in the returns of a specific market benchmark or index.[137]

ERP = the equity risk premium. The ERP is the rate of return added to a risk-free rate to reflect the additional risk of equity securities over risk-free securities.

Research tells us that the CAPM often misprices risk for certain investments. Specifically, researchers have observed that commonly used methods of measuring risk used in the CAPM (specifically, beta) often understate the risk (and thus understate the required return) for small company stocks. Examination of market evidence shows that within the context of CAPM, beta does not fully explain the difference between small company returns and large company returns. In other words, the historical (observed) excess return of portfolios comprised of smaller companies is greater than the excess return predicted by the CAPM for these portfolios. This "premium over CAPM" is commonly known as a "beta-adjusted size premium" or simply "size premium".[138]

It follows that the size premium is a necessary correction to the basic CAPM because risk, as measured by the betas of smaller companies (even sum betas), is systematically underestimated.[139] Moreover, the size effect is not just evident for the smallest companies in the marketplace, but is evident for all but the largest groups of companies, including companies with a market capitalization in excess of $1 billion. A common practice is to incorporate this evidence by adding a size premium to the CAPM formula when valuing companies that are comparatively small. The modified CAPM formula is[140]:

$$COE_{CAPM} = R_f + (ß \ x \ ERP) + RP_s$$

where:

RP_s = the "beta-adjusted" size premium.

It is important to note that the risk premia over CAPM (i.e. "size premia") published in the *Risk Premium Report* are adjusted for beta.[141] In other words, the portion of excess return that is not attributable to beta is controlled for, or removed, leaving only the size effect's contribution to excess return. These premia are appropriate for use in the capital asset pricing model (CAPM), and in buildup methods that do not otherwise *already* have a measure of size risk.[142]

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[136] John R. Graham and Campbell R. Harvey, "The Theory and Practice of Corporate Finance," *Journal of Financial Economics* (May 2001): 187–243.

[137] For the purposes of this report, the "market" is defined as the S&P 500 Index. The S&P 500 Index is a broad-based, market-capitalization-weighted index widely regarded as being representative of the overall market.

[138] A sample of academic research articles include: Rolf Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981): 3–18; Eugene Fama and Kenneth French, "The Cross Section of Expected Stock Returns," *Journal of Finance* (June 1992): 427–486; Kent Daniel and Sheridan Titman, "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns," *Journal of Finance* (March 1997): 1–33.

[139] The betas presented in the *Risk Premium Report* are "sum" betas. Smaller companies generally trade more infrequently and exhibit more of a "lagged" price reaction (relative to the market) than do large stocks. One of the ways of capturing this lag movement is called sum beta. See Ibbotson, Roger G., Paul D. Kaplan, and James D. Pearson. "Estimates of Small-Stock Betas Are Much Too Low," *Journal of Portfolio Management*, Summer 1997.

[140] A "modified CAPM" typically refers to the common modification to the CAPM formula that is used to incorporate an adjustment for size.

[141] For a detailed discussion of how premia over CAPM ("size premia") are calculated, see "The Difference Between 'Risk Premia Over the Risk-Free Rate' and 'Risk Premia Over CAPM'" on page 43.

[142] For example, the size premia presented in Exhibit B cannot be used in "Buildup 1". The Buildup 1 method uses "risk premia over the risk-free rate" (from Exhibit A) that *already* have a measure of risk in terms of the combined effect of market risk and size risk, (RP_{m+s}). Using size premia in Buildup 1 would be "double counting" size risk.

The Size Study

Please note that base estimates of COE developed with the modified CAPM equation presented above are *after* addition of a size premium, but *prior* to the addition of any company-specific risk premiums (RP_u) that the individual analyst may deem to be applicable.[143] Company-specific risk can be added by the individual analyst to the modified CAPM in the following fashion:

$$COE_{CAPM} = R_f + (ß \; x \; ERP) + RP_s + RP_u$$

The *Risk Premium Report* provides two ways for analysts to match their subject company's size (or risk) characteristics with the appropriate smoothed premia from the data exhibits: the "guideline portfolio" method and the "regression equation" method.[144] In general, the regression equation method is preferred because this method allows for interpolation between the individual guideline portfolios, although the guideline portfolio method is less complicated, and more direct.

Examples of both the guideline portfolio method and the regression equation method follow, starting with the simpler guideline portfolio method.

Example 3a: CAPM Method (using guideline portfolios)

Four pieces of information are needed to estimate the cost of equity capital using the CAPM method and "guideline portfolios": a risk-free rate (R_f), a beta ($ß$), an equity risk premium (ERP), and a risk premium over CAPM (RP_s, otherwise known as a beta-adjusted "size premium"). All of the information needed is summarized in Figure 30.

Figure 30: Information Needed to Estimate COE Using CAPM and Guideline Portfolios
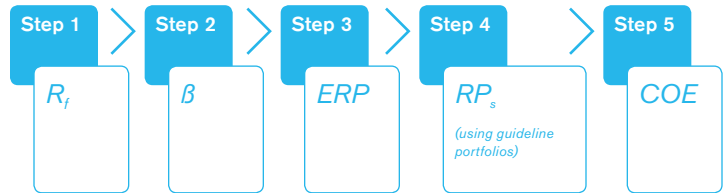
This example utilizes the risk-free rate (R_f) and ERP that were established in Example 1a (see page 51). This mirrors the fact that for any given valuation engagement the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. For any given valuation engagement these inputs may be (and probably will be) different than the ones used in the examples.

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[143] The "u" in RP_u stands for underscore{u}nique risk or company-specific risk, based upon the so-called "company-specific" differences of the subject company's fundamental risk relative to the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.
[144] See page 19 for a detailed explanation of the differences between the guideline portfolio method and the regression equation method.

The Size Study

Step 1, Risk-Free Rate (R_f): The risk-free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the normalized long-term treasury yield of 4.0 percent established in Example 1a (page 51).

Step 2, Beta (β): Beta is a measure of the sensitivity of changes in the returns (dividends plus price changes) of a stock relative to changes in returns of a specific market benchmark or index. Duff & Phelps does not currently publish company betas or peer group betas.[145] Because the sum betas calculated for the 25 size-ranked portfolios in the B exhibits are betas for a particular *size* of company, they would in all likelihood not be appropriate for use within a CAPM estimate of COE. Betas for use within a CAPM estimate of COE are more appropriately estimated by utilizing publicly-traded company data to directly estimate betas for the *industry* (or smaller *industry* peer group) that the company operates in, rather than using betas that are primarily based upon "size". For this example, a beta of 1.2 is assumed.

Step 3, Equity Risk Premium (ERP): The ERP is the rate of return added to a risk-free rate to reflect the additional risk of equity instruments over risk-free instruments. For this example, the Duff & Phelps Recommended ERP as of the end of 2012 (5.5%) is assumed.[146, 147]

Step 4, Risk Premium Over CAPM ("size premium") (RP_s): Match the various size measures of the subject company with the guideline portfolios composed of companies of similar size in Exhibits B-1 through B-8, and identify the corresponding smoothed average risk premium over CAPM (i.e. "size premium").

The subject company in this example has a market value of equity of $120 million, and the appropriate data exhibit is Exhibit B-1 (see Figure 11 on page 48). An abbreviated version of Exhibit B-1 is shown in Figure 31. Of the 25 portfolios, the portfolio that has an average market value *closest* to the subject company's $120 million market value is portfolio 25 ($94 million). The corresponding smoothed average size premium is 7.55 percent (7.5 percent, rounded).

Match each of the subject company's size measures in this fashion. For example, the second size measure for the subject company in this example is "book value of equity" of $100 million. Of the 25 guideline Portfolios in Exhibit B-2 (not shown here), the portfolio that has an average book value of equity *closest* to the subject company's $100 million book value is portfolio 25 ($67 million). The corresponding smoothed average size premium is therefore 6.0 percent. After all of the available size measures for the subject company have been matched to the closest guideline portfolio in the appropriate exhibit and the corresponding smoothed average size premium has been identified for each, Step 4 is complete.

Figure 31: Exhibit B-1 (abbreviated)

Companies Ranked by Market Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Mkt Value (in $millions)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	*Smoothed Premium over CAPM*
1	136,859	5.14	0.83	11.72%	4.96%	3.71%	1.26%	*-1.14%*
2	39,247	4.59	0.95	10.54%	3.79%	4.24%	-0.45%	*0.35%*
3	25,711	4.41	0.93	11.11%	4.36%	4.15%	0.20%	*0.85%*
				///				
24	288	2.46	1.25	18.91%	12.16%	5.55%	6.60%	*6.21%*
25	94	1.97	1.28	22.70%	15.95%	5.71%	10.24%	*7.55%*

[145] Company betas and industry betas are available from multiple sources, including Morningstar (www.morningstar.com), Bloomberg (www.bloomberg.com), MSCI (www.MSCI.com), and Value Line (www.valueline.com).
[146] For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.
[147] For more information on cost of capital issues, including developing risk-free rates and ERP during periods of flight to quality, please visit www.DuffandPhelps.com/CostofCapital

The Size Study

Step 5, Estimate Cost of Equity (COE): With the completion of Steps 1 through 4, the information needed to estimate a base cost of equity capital using the CAPM is now completed. The risk premia over CAPM (RP_s or "size premia") can now be added to the basic CAPM equation ($COE_{CAPM} = R_f + (\beta \times ERP) + RP_s$) to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 32.

The range of cost of equity capital estimates for the hypothetical subject company in this example is 16.1 percent to 18.1 percent, with an average of 17.0 percent, and a median of 16.9 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" estimate in the group.

Remember that the full CAPM equation is:

$$COE_{CAPM} = R_f + (\beta \times ERP) + RP_s + RP_u$$

The base cost of equity capital estimates derived in this example are therefore *prior* to the addition of any other company-specific risk premiums (RP_u) that the individual analyst may deem appropriate.

Figure 32: CAPM COE Inputs (using guideline portfolios)

	Size Measure (in $millions)	Appropriate Exhibit	Guideline Portfolio	Step 1 Risk-Free Rate, R_f		Step 2 Beta β		ERP		Step 4 Smoothed Premium Over CAPM (size premium), RP_s		Step 5 COE*
Market Value of Equity	$120	B-1	25	4.0%	+	(1.2	x	5.5%)	+	7.5%	=	18.1%
Book Value of Equity	$100	B-2	25	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
5-year Average Net Income	$10	B-3	24	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
Market Value of Invested Capital	$180	B-4	25	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
Total Assets	$300	B-5	24	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
5-year Average EBITDA	$30	B-6	25	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
Sales	$250	B-7	24	4.0%	+	(1.2	x	5.5%)	+	5.5%	=	16.1%
Number of Employees	200	B-8	25	4.0%	+	(1.2	x	5.5%)	+	░░░░	=	░░░░
		Mean (average) values		4.0%	+	(1.2	x	5.5%)	+	6.4%	=	17.0%
		Median (typical) values		4.0%	+	(1.2	x	5.5%)	+	6.3%	=	16.9%

Note: Some values intentionally blurred.

* Difference(s) due to rounding.

The Size Study

Example 3b: CAPM Method (using regression equations)

When the subject company size measures do not exactly match the respective average company size of the guideline portfolios, the data exhibits provide a straightforward way to interpolate an "exact" risk premium over CAPM between guideline portfolios using the "regression equation" method.

The *only* difference between estimating cost of equity capital (COE) using the CAPM method using "guideline portfolios" (as in the previous example) and estimating cost of equity capital using the CAPM method using "regression equations" is how the risk premia over CAPM (RP_s or "size premia") are identified in Step 4.

Figure 33: Information Needed to Estimate COE Using CAPM and Regression Equations

Step 1		Step 2		Step 3		Step 4		Step 5
R_f	>	$ß$	>	ERP	>	RP_s *(using regression equations)*	>	COE

In the previous example, the smoothed average risk premia over CAPM *published* in the *Report* for the appropriate guideline portfolios were used to estimate COE.[148] In this example, however, the regression equations found in each of the data exhibits will be used to calculate "custom" interpolated size premia, based upon the specific size measures of the subject company.

This example utilizes the long-term risk-free rate (R_f) and ERP established in a previous example (the *Size Study*'s Buildup 1 method using "guideline portfolios"; see page 51), and the Beta ($ß$), established for the previous example (Example 3a on page 69). This mirrors the fact that for any given valuation engagement the same inputs will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may be (and probably will be) different than the ones used in the examples. The only missing ingredients needed to estimate cost of equity capital are the premia over CAPM, or size premia (RP_s), as summarized in Figure 34.

Figure 34: Needed—Smoothed Premia Over CAPM (RP_s, or "Size Premia") Calculated Using Regression Equations

	Size Measure (in $millions)	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Beta $ß$		Step 3 ERP		Step 4 Smoothed Premium Over CAPM (size premium), RP_s	Step 5 COE
Market Value of Equity	$120	B-1	4.0%	+	(1.2	x	5.5%)	+	?	=
Book Value of Equity	$100	B-2	4.0%	+	(1.2	x	5.5%)	+	?	=
5-year Average Net Income	$10	B-3	4.0%	+	(1.2	x	5.5%)	+	?	=
Market Value of Invested Capital	$180	B-4	4.0%	+	(1.2	x	5.5%)	+	?	=
Total Assets	$300	B-5	4.0%	+	(1.2	x	5.5%)	+	?	=
5-year Average EBITDA	$30	B-6	4.0%	+	(1.2	x	5.5%)	+	?	=
Sales	$250	B-7	4.0%	+	(1.2	x	5.5%)	+	?	=
Number of Employees	200	B-8	4.0%	+	(1.2	x	5.5%)	+	?	=
		Mean (average) values	4.0%	+	(1.2	x	5.5%)	+		=
		Median (typical) values	4.0%	+	(1.2	x	5.5%)	+		=

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[148] The smoothed risk premia published in the *Risk Premium Report* are based upon the average size (or risk) measure in each of the respective guideline portfolios.

The Size Study

Step 1, Risk-Free Rate (R_f): The risk-free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the normalized long-term treasury yield of 4.0 percent used in Example 1a (page 51).

Step 2, Beta (β): Beta is a measure of the sensitivity of a stock's price relative to movements of a specific market benchmark or index. For this example, the beta of 1.2 that was assumed in Example 3a (page 69) is assumed.

Step 3, Equity Risk Premium (ERP): The ERP is the rate of return added to a risk-free rate to reflect the additional risk of equity instruments over risk-free instruments. For this example, the Duff & Phelps Recommended ERP as of the end of 2012 (5.5%) is assumed.[149, 150]

Step 4, Risk Premium Over CAPM (RP_s): The hypothetical subject company in this example has a market value of equity of $120 million, and the appropriate *Size Study* data exhibit to use is Exhibit B-1[151]. In this case one would expect that the smoothed average premium over CAPM, or size premium, would fall somewhere between 6.21 percent (the smoothed size premium for Portfolio 24) and 7.55 percent (the smoothed size premium for Portfolio 25), as illustrated in Figure 35:

An easy way to calculate a custom interpolated risk premium over CAPM (RP_s or "size premia") "in between" Portfolio 24 and Portfolio 25 is by using the regression equations provided for this purpose in each of the data exhibits. The regression equations are located in the same spot in each of the exhibits (see Figure 7 on page 20).[152]

Figure 35: Exhibit B-1 (abbreviated)

Companies Ranked by Market Value of Equity
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank by Size	Average Mkt Value (in $millions)	Log of Size	Beta (SumBeta) Since '63	Arithmetic Average Return	Arithmetic Average Risk Premium	Indicated CAPM Premium	Premium over CAPM	Smoothed Premium over CAPM (RP_s)
1	136,859	5.14	0.83	11.72%	4.96%	3.71%	1.26%	*-1.14%*
2	39,247	4.59	0.95	10.54%	3.79%	4.24%	-0.45%	*0.35%*
3	25,711	4.41	0.93	11.11%	4.36%	4.15%	0.20%	*0.85%*
				///				
24	288	2.46	1.25	18.91%	12.16%	5.55%	6.60%	**6.21%**
Subject Company	**120**							**?**
25	94	1.97	1.28	22.70%	15.95%	5.71%	10.24%	**7.55%**

[149] For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.

[150] For more information on cost of capital issues, including developing risk-free rates and ERP during periods of flight to quality, please visit www.DuffandPhelps.com/CostofCapital

[151] The same eight size measures (for a hypothetical subject company) are used in *all* examples of estimating COE using the *Size Study*, as outlined in Figure 11 on page 48.

[152] In addition to regression equations for interpolating risk premia between guideline portfolios in the *Size Study*'s A and B exhibits, the *Risk Study*'s D exhibits also provide regression equations for easy interpolation of risk premia between guideline portfolios, as do the C exhibits (for unlevered risk premia).

The Size Study

The regression equation provided in Exhibit B-1, which includes 25 portfolios ranked by market value[153], is:

*Smoothed Premium = 12.971% − 2.748% * Log (Market Value)*

To calculate an interpolated smoothed risk premium over CAPM (RP_s or "size premia") for the subject company's $120 million market value, substitute the market value into the regression equation as follows[154]:

*Smoothed Premium= 12.971% − 2.748% * Log (120)*

*7.3% = 12.971% − 2.748% * 2.08*

Continue interpolating smoothed risk premium over CAPM for each size measure available for the subject company using the regression equations from the data exhibits. For example, the second size measure for the subject company is "book value of equity" of $100 million. The equation found on Exhibit B-2 is:

*Smoothed Premium= 9.320% − 1.810% * Log (Book Value)*

The interpolated smoothed risk premium over CAPM is therefore 5.7 percent (9.320% − 1.810% * 2). After interpolating smoothed size premia for all of the subject company's available size measures, Step 4 is complete, as shown in Figure 36.

Figure 36: Calculation of Smoothed Risk Premia Over CAPM (RP_s) Using Regression Equations

Appropriate Exhibit	Size Measure	Subject Company Size Measures (in $millions)	Appropriate Regression Equation		Step 4 Smoothed Risk Premium Over CAPM (size premium), RP_s
B-1	Market Value of Equity	$120	Smoothed Premium = 12.971% − 2.748% * Log(Market Value)	=	*7.3%*
B-2	Book Value of Equity	$100	Smoothed Premium = 14.80% − 14.80% * Log(Book Value)	=	14.80%
B-3	5-year Average Net Income	$10	Smoothed Premium = 14.80% − 14.80% * Log(Net Income)	=	14.80%
B-4	Market Value of Invested Capital	$180	Smoothed Premium = 14.80% − 14.80% * Log(MVIC)	=	14.80%
B-5	Total Assets	$300	Smoothed Premium = 14.80% − 14.80% * Log(Invested Capital)	=	14.80%
B-6	5-year Average EBITDA	$30	Smoothed Premium = 14.80% − 14.80% * Log(EBITDA)	=	14.80%
B-7	Sales	$250	Smoothed Premium = 14.80% − 14.80% * Log(Sales)	=	14.80%
B-8	Number of Employees	200	Smoothed Premium = 14.80% − 14.80% * Log(Employees)	=	14.80%

Note: Some values intentionally blurred.

[153] Figure 11 on page 48 lists the appropriate "B" exhibits in which the size premia for each of the eight size measures can be found.

[154] Please note that the logarithmic relationship is base-10, and that the financial size data is in millions of dollars, such that the log of $10 million is log (10), not log (10,000,000). The formula to calculate a value's base-10 logarithm in Microsoft Excel is "=log (value)". The "*" used in the regression equation is the symbol used in Microsoft Excel to denote the multiplication symbol, "x". The "*" format is used to denote multiplication in the regression equations in the data exhibits.

The Size Study

Step 5, Estimate Cost of Equity (COE): With the completion of Steps 1 through 4, the information needed to estimate a base cost of equity capital using the CAPM (using regression equations) is now completed. The risk premiums over CAPM (RP_s or "size premia") can now be added to the basic CAPM equation ($COE_{CAPM} = R_f + (ß \times ERP) + RP_s$) to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 37.

The range of cost of equity capital estimates for the hypothetical subject company in this example is 16.2 percent to 17.9 percent, with an average of 16.9 percent, and a median of 16.8 percent. The mean estimate is the simple average of the COE estimates, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median cost of equity capital estimate is generally preferred to the mean. The median tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" COE estimate in the group.

Remember that the full CAPM equation is:

$$COE_{CAPM} = R_f + (ß \times ERP) + RP_s + RP_u$$

The base cost of equity capital estimates derived in this example are therefore *prior* to the addition of any company-specific risk premiums (RP_u) that the individual analyst may deem appropriate.

Figure 37: CAPM COE Inputs (using regression equations)

	Size Measure (in $millions)	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Beta ß		Step 3 ERP		Step 4 Smoothed Premium Over CAPM (size premium), RP_s		Step 5 COE*
Market Value of Equity	$120	B-1	4.0%	+	(1.2	x	5.5%)	+	7.3%	=	17.9%
Book Value of Equity	$100	B-2	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
5-year Average Net Income	$10	B-3	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
Market Value of Invested Capital	$180	B-4	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
Total Assets	$300	B-5	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
5-year Average EBITDA	$30	B-6	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
Sales	$250	B-7	4.0%	+	(1.2	x	5.5%)	+	5.6%	=	16.2%
Number of Employees	200	B-8	4.0%	+	(1.2	x	5.5%)	+	14.80%	=	14.80%
		Mean (average) values	4.0%	+	(1.2	x	5.5%)	+	6.3%	=	16.9%
		Median (typical) values	4.0%	+	(1.2	x	5.5%)	+	6.2%	=	16.8%

Note: Some values intentionally blurred.

* Difference(s) due to rounding.

The Risk Study

The *Risk Study* is an extension of the *Size Study*. The main difference between the *Risk Study* and the *Size Study* is that while the *Size Study* analyzes the relationship between size and return, the *Risk Study* analyzes the relationship between fundamental risk measures (based on accounting data) and return. These are called "fundamental" measures of company risk to distinguish these risk measures from a stock market based measure of equity risk such as beta. A variety of academic studies have examined the relationship between financial statement data and various aspects of business risk.[160] Research has shown that measures of earnings volatility can be useful in explaining credit ratings, predicting bankruptcy, and explaining the CAPM beta.

As in the *Size Study*, 25 portfolios are created, but instead of being ranked by eight alternative measures of size as is done in the *Size Study*, the *Risk Study* portfolios are ranked by three fundamental risk measures: five-year average operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity.[161, 162] The first statistic measures profitability and the other two statistics measure volatility of earnings. All three measures use average financial data for the five years preceding the formation of annual portfolios.

Size and Risk

Traditionally, valuation professionals have used company size as a factor in determining discount rates for smaller companies. Small companies are believed to have higher required rates of return than large companies because small companies are inherently riskier. The historical data (as published in the *Risk Premium Report*, as well as in the *SBBI Yearbook*), verify that small companies have, in fact, earned higher rates of return over long-run periods.

It has been pointed out in the financial literature that researchers may be mixing a "size" effect with a "risk" effect when measuring company size by market value,[163] but market value is not just a function of "size"; it is also a function of the discount rate. In other words, some companies might be small because they are risky, rather than risky because they are small. The *Risk Study* goes beyond size and investigates the relationship between equity returns and fundamental risk measures. Does the evidence support the claim that smaller companies inherently have greater risk? The *Risk Study* analyzes this question, and demonstrates that as company size decreases, measures of risk calculated from financial statement data do, as a matter of fact, tend to increase. The data clearly shows that as fundamental risk *increases* in the form of lower profitability or greater variability of earnings, the return over the risk-free rate tends to *increase*. These relationships are summarized in Figure 42.

Figure 42: Operating Margin (i.e. "profitability") and Variability of Earnings versus Risk.

Operating Margin — Risk   Variability of Earnings — Risk

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[160] A survey of the academic research can be found in *The Analysis and Use of Financial Statements*, 3rd edition, White et al., Wiley (2003), chapter 18.

[161] Coefficient of variation is defined here as the standard deviation divided by the mean.

[162] For a detailed discussion of portfolio creation methodology, see "Portfolio Methodology" on page 6.

[163] "A Critique of Size Related Anomalies," Jonathan Berk, *Review of Financial Studies*, vol. 8, no. 2 (1995).

The Risk Study

Previously, it was demonstrated in the *Size Study* that there is a clear *inverse* relationship between size and historical rates of return (as size decreases, returns tend to increase; see Graph 4 on page 25). In the *Risk Study*, the data show a clear *direct* relationship between accounting-data-based fundamental risk measures and historical rates of return (as fundamental risk increases, returns tend to increase).

In Graph 16, as fundamental risk *increases* (from left to right), average annual return over the study time horizon (1963–2012) tends to *increase* for each of the three fundamental risk measures.

For example, in the *2013 Report*, the average annual return of the portfolios made up of companies with the lowest risk as measured by each of the three fundamental risk measures was 13.2 percent, while the average annual return of the portfolios made up of companies with the highest risk as measured by each of the three fundamental risk measures was 20.1 percent.

Reasons for Using Fundamental Measures of Risk in Addition to Measures of Size

First, certain measures of size (such as market value of equity) may be imperfect measures of the risk of a company's operations in some situations. For example, a company with a large and stable operating margin may have a small and unstable market value of equity if it is highly leveraged. In this case the risk of the underlying operations is *low* while the risk to equity is *high*.

Second, while small size may indicate greater risk, some small companies may maintain near economic monopolies by holding a geographic niche or market niche such that their true riskiness is *less* than what would be indicated by their size.

Graph 16: Average Annual Return, Three Measures of Fundamental Risk
1963–2012
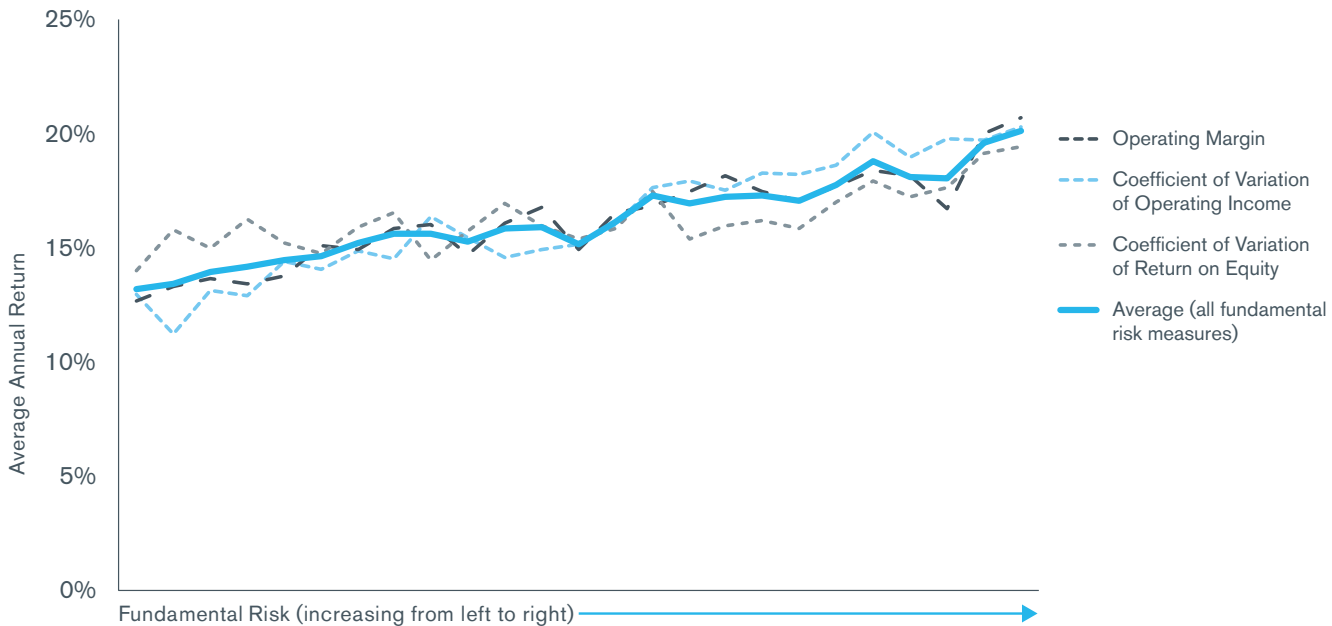
The Risk Study

Alternatively, while larger size (as measured by sales, for example) may indicate less risk, some companies may be riskier than the average of companies with similar sales. For example, assume the subject company was expecting to emerge from reorganization following bankruptcy. The risk premium appropriate for this company may be more accurately imputed from the pro-forma operating profit (after removing non-recurring expenses incurred during the bankruptcy) than from its size as measured by sales. In other words, the subject company may be riskier than companies with similar sales volumes.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the risk premium based on size measures, this difference may be an indication of the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.[164]

The "D" Exhibits – Summary of Data Presented

The *Risk Study*'s D exhibits present 25 portfolios ranked by three fundamental risk factors (based on accounting data). These fundamental risk factors are described in Table 7.[165]

Table 7: Three Measures of Fundamental Risk in the *Risk Study*'s D Exhibits

Exhibits D-1
Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation. Note that this composite ratio is usually very close to a simple average of the annual ratios of operating income to sales, except in extreme cases generally involving companies with high growth rates.

Exhibit D-2
Coefficient of Variation of Operating Margin: The standard deviation of operating margin over the prior five years divided by the average operating margin for the same years. Note that for calculating this coefficient, average operating margin is a simple average of the annual ratios of operating income to sales rather than the composite ratio used in Exhibit D-1.

Exhibit D-3
Coefficient of Variation of Return on Book Value of Equity: The standard deviation of return on book equity for the prior five years divided by the mean return on book equity for the same years. Return on book equity is defined as net income before extraordinary items minus preferred dividends divided by book value of common equity.

Each of the *Risk Study*'s Exhibits D-1 through D-3 displays one line of data for each of the 25 fundamental-risk-ranked portfolios. The D exhibits include the *statistics* outlined in Table 8 for *each* of the *risk measures* outlined in Table 7.

For comparative purposes, the average returns from the *SBBI* series for large companies (essentially the S&P 500 Index), small companies, and long-term government bond income returns for the period 1963 through the latest year are also reported on each exhibit.[166]

Table 8: Statistics Reported for 25 fundamental-risk-ranked portfolios in the *Risk Study*'s D Exhibits

• The average of the sorting criteria for the latest year (e.g., the average operating margin for the latest five years before 2012). In the *2013 Report*, the "latest year" is 2012. Note that the reported average risk statistics in Exhibits D-1, D-2, and D-3 are not the same numbers as reported in Exhibits C-1 through C-8. In Exhibits C-1 through C-8, the reported statistics are calculated for portfolios of companies grouped according to size and are averages since 1963. In Exhibits D-1, D-2, and D-3, the reported statistics are calculated for portfolios grouped according to risk, independent of the "size" of the companies, and are not averages since 1963	• Geometric average historical equity return since 1963.
• Log (base-10) of the average of the sorting criteria.	• Arithmetic average historical equity return since 1963.
• The number of companies in each portfolio in the latest year. In the *2013 Report*, the "latest year" is 2012.	• Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. (RP_{m+u})
• Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).	• Unlevered arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. ($RP_{m+u, unlevered}$)
• Unlevered beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year.	• "Smoothed" average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963: the fitted premium from a regression with the historical "risk premium over long-term Treasuries" as dependent variable and the logarithm of the average sorting criteria as independent variable. (RP_{m+u})
• Standard deviation of annual historical equity returns.	• Average Debt as a percent of the MVIC since 1963.

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[164] *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Examples of risks that are typically referred to as "company-specific" risk can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are perceived as unique to the subject company.

[165] For a detailed description of the Standard and Poor's *Compustat* data items used in the *Duff & Phelps Risk Premium Report*, please see Appendix A.

[166] Source: Morningstar *EnCorr* software.

The Risk Study

Is Size Correlated with Market and Fundamental Risk Measures?

It is important to understand that the 25 portfolios used to calculate the fundamental risk statistics included in the D exhibits are *different* from the 25 portfolios used to calculate the fundamental risk statistics included in the C exhibits. In the latter case, the portfolios are ranked by each of eight alternative measures of size, and then the fundamental risk characteristics of each portfolio are calculated. In the former case, the large base set of companies that the analyses of the *Report* begins with are ranked by each of the three fundamental risk measures to form 25 *risk-ranked* portfolios, and *then* the average risk characteristics of each portfolio are calculated. For example, if 10 companies were ranked by size, the order (from largest to smallest) may be quite different from the same 10 companies ranked by operating margin (from highest to lowest).[167]

However, the data suggests that size is correlated with market measures. For example, as size measures decrease in Graph 17 (from left to right), the beta (both levered and unlevered) of the portfolios increase (as expected).[168]

Graph 17: Average Levered and Unlevered Sum Beta (all eight size measures)
1963–2012

Portfolio (1 = Largest, 25 = Smallest)

- - - Average Levered Sum Beta (all eight size measures)
⎯⎯ Average Unlevered Sum Beta (all eight size measures)

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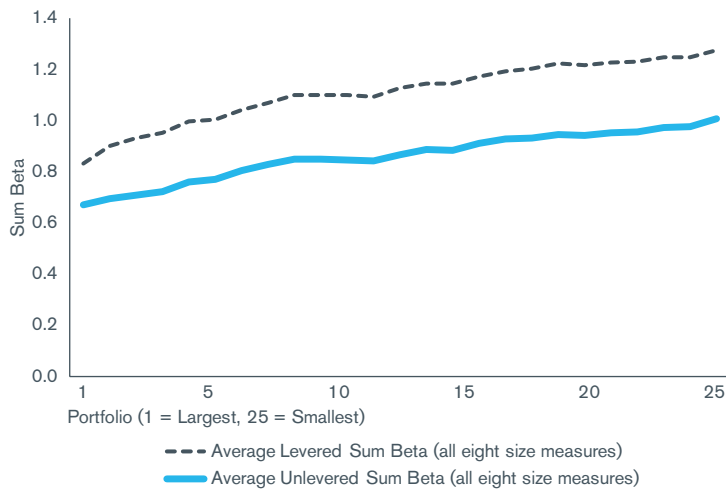
[167] For more information on the "C" Exhibits, see page 112.

[168] In the research on "size" as reported in this report, we have determined that, in the context of the CAPM, the higher betas of the small companies explain some but not all of the higher average historical equity returns in these portfolios.

The Risk Study

The data also suggests that this correlation extends to the three fundamental measures of risk. For example, in Graph 18(a), as size measures decrease (from left to right), operating margin of the portfolios decreases (indicating increased risk), and in Graph 18(b), as size measures decrease (from left to right), average coefficient of operating margin and average coefficient of variation of ROE of the portfolios increase (indicating increased risk).

While the correlation between fundamental measures of risk and size clearly demonstrated in Graph 18(a) and Graph 18(b) implies that there may be an embedded "size effect" component in the *Risk Study*'s RP_{m+u} premia, the magnitude of this embedded size effect is difficult to quantify. In any case, the size effect embedded in the *Risk Study*'s RP_{m+u} premia are in all likelihood not equivalent to the size effect embedded in the *Size Study*'s RP_{m+s} premia, which are a measure of risk in terms of the combined effect of *market risk* and *size risk*.

To avoid confusion between the two premia, and because the operating efficiencies (or lack thereof) of the subject company being captured by the use of accounting-based risk measures may offset the risk premium resulting from the size effect (or increase the risk premium resulting from the size effect), the *Report* characterizes the *Risk Study*'s "risk premia over the risk-free rate" (RP_{m+u}) as being a measure of risk in terms of the combined effect of *market risk* and *company-specific risk* (also known as "unsystematic risk").

Generally, the three fundamental measures of risk display increasing risk as size decreases, as the historical unlevered risk premium increases and as the unlevered beta increases.[169, 170]

Graph 18(a): Average Operating Margin (all eight size measures) 1963–2012

- Average Operating Margin (all eight size measures)
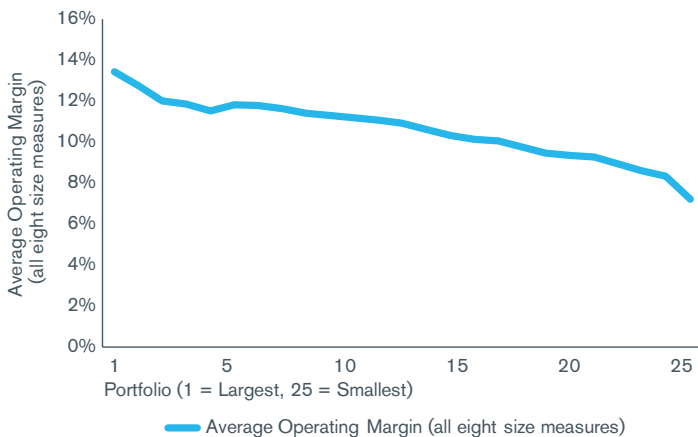
Graph 18(b): Average Coefficient of Operating Margin and Average Coefficient of Variation of ROE (all eight size measures) 1963–2012

- - - Average CV(ROE) (all eight size measures)
- Average Operating Margin (all eight size measures)

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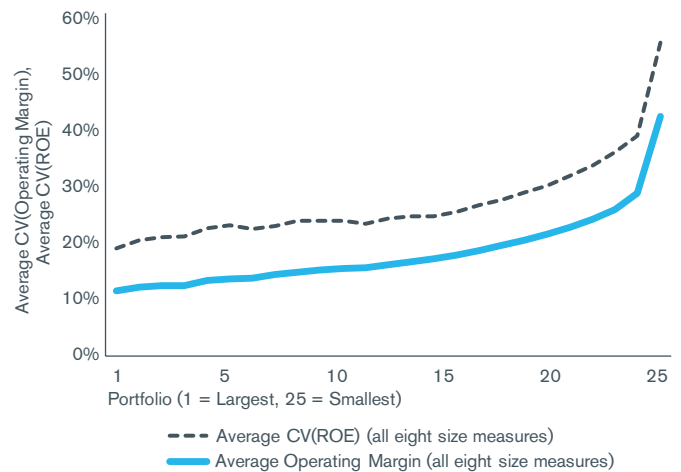
[169] Were one to calculate the respective correlations, those statistics would relate average portfolio statistics (e.g. average size vs. average risk) rather than correlation statistics across individual companies. At the individual company level, the correlations are much lower.

[170] There are two notable exceptions to this pattern: Exhibit C-7 indicates that there is little differentiation in operating margin as size (as measured by sales) changes, and Exhibit C-8 indicates that there is little differentiation in operating margin as size (as measured by number of employees) changes. In both cases, however, the coefficient of variation of operating margin and the coefficient in variation of return on book equity indicate increasing risk as size (as measured by sales and number or employees) decreases, as in the other exhibits.

The Risk Study

The Buildup 3 method for estimating the cost of equity capital using the *Report's* Risk Study is shown in this excerpted version of the *Report*.

Examples for each of the methods of cost of equity capital estimation methods available using the *Report* (see Table 1) are available in the full version *Report*.

Overview of Methods Used to Estimate Cost of Equity Capital using the Risk Study

The *Risk Study* provides one method of estimating cost of equity capital for a subject company, Buildup 3, plus one method for estimating unlevered cost of equity capital (the cost of equity capital assuming a firm is financed 100% with equity and 0% debt).

These methods are summarized below in equation format, and summarized in Figure 43 in graphical "building blocks" format.

1) **Buildup 3**
   $COE_{Buildup\ 3}$ = (Risk-Free Rate) + (Risk Premium Over Risk-Free Rate) + (Equity Risk Premium Adjustment)

   Example 5a: using guideline portfolios: page 89
   Example 5b: using regression equations: page 92

2) **Buildup 3-Unlevered**
   $COE_{Buildup\ 3\text{-}Unlevered}$ = (Risk-Free Rate) + (Unlevered Risk Premium Over Risk-Free Rate) + (Equity Risk Premium Adjustment)

   Example 6: using Guideline portfolios: page 96

Figure 43: Two Methods of Estimating Cost of Equity Capital with the *Risk Study*[171]
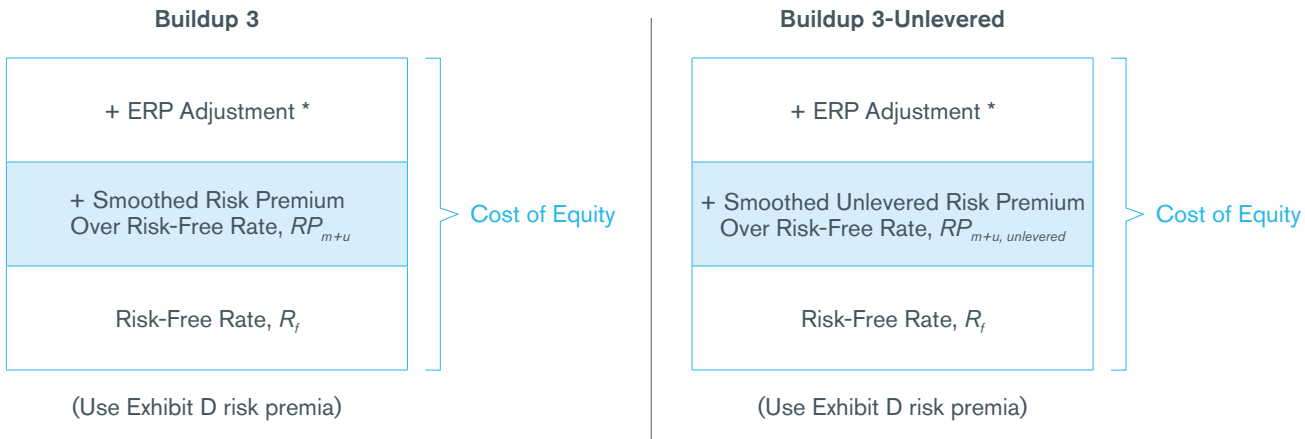
Buildup 3	Buildup 3-Unlevered
+ ERP Adjustment *	+ ERP Adjustment *
+ Smoothed Risk Premium Over Risk-Free Rate, RP_{m+u}	+ Smoothed Unlevered Risk Premium Over Risk-Free Rate, $RP_{m+u,\ unlevered}$
Risk-Free Rate, R_f	Risk-Free Rate, R_f
(Use Exhibit D risk premia)	(Use Exhibit D risk premia)

*ERP Adjustment: The difference between the historical (1963–2012) equity risk premium (ERP) and a user of the *Duff & Phelps Report*'s own forward ERP estimate:

ERP Adjustment = User's ERP – Historical (1963–2012) ERP

The ERP Adjustment is made only in the "Buildup 1", "Buildup1-Unlevered", "Buildup 1-High-Financial-Risk", "Buildup 3", and "Buildup 3-Unlevered" methods. Please refer to the individual examples provided for these models for more information. For a detailed discussion of the ERP Adjustment, see page 12.

[171] The relative sizes of the "building blocks" in Figure 43 do not necessarily represent the relative size of the various inputs. Also note that the names given to the models in the *Risk Premium Report* (e.g. "Buildup 1", "Buildup 2", "Buildup 3", etc) are naming conventions used within the *Report* to make referring to the different methods easier.

The Risk Study

The three risk measures outlined in Table 7 (page 82) can be used with either of the two methods of estimating COE provided by the *Risk Study*. It is important to note that the subject company information necessary to calculate all of these measures may not be available. In these cases, it is generally acceptable to use the fundamental risk measures that *are* available. It is recommended, however, that *Report* users calculate available risk measures for the subject company using at least the three most recent years of data, and the five most recent years of data for best results.

Gathering Accounting Information to Calculate Fundamental Risk Measures

The first step in using the *Risk Study* to estimate cost of equity capital (COE) is to gather the accounting-based information for the subject company needed to calculate the three fundamental risk measures analyzed in the *Risk Study*.

- To calculate "operating margin" and "coefficient of variation of operating margin", net sales and operating income are needed.

- To calculate "coefficient of variation of ROE", book value and net income before extraordinary items are needed.

The accounting information for the last 5 years needed to calculate the three fundamental risk measures for a hypothetical subject company is summarized in Figure 44(a) and Figure 44(b).

Figure 44(a): Subject Company Operating Margin and Coefficient of Variation of Operating Margin (used in all examples)

	2012	2011	2010	2009	2008
Net Sales	$900	$800	$850	$750	$900
Operating Income	$150	$120	$130	$80	$140
Operating Margin	16.7% = $150/$900	15.0% = $120/$800	15.3% = $130/$850	10.7% = $80/$750	15.6% = $140/$900
Standard Deviation of Operating Margin	2.3%				
Average Operating Margin	**14.6%**				
Coefficient of Variation of Operating Margin	**15.8%** = 2.3%/14.6%				

Figure 44(b): Subject Company Coefficient of Variation of ROE (used in all examples)

	2012	2011	2010	2009	2008
Book Value	$820	$710	$630	$540	$500
Net Income before extraordinary items	$110	$80	$90	$40	$100
Return on Book Equity (ROE)	13.4% = $110/$820	11.3% = $80/$710	14.3% = $90/$630	7.4% = $40/$540	20.0% = $100/$500
Standard Deviation of ROE	4.6%				
Average ROE	13.3%				
Coefficient of Variation of ROE	**34.7%** = 4.6%/13.3%				

The Risk Study

The hypothetical subject company has an average operating margin of 14.6 percent, a coefficient of variation of operating margin of 15.8 percent, and a coefficient of variation of ROE of 34.7 percent, as summarized in Figure 45.[172]

Figure 45: Subject Company Fundamental Risk Characteristics (used in all Examples)

		Appropriate Exhibit	
	Risk Measure	Buildup 3	Buildup 3-Unlevered
Average Operating Margin	14.6%	D-1	D-1
Coefficient of Variation of Operating Margin	15.8%	D-2	D-2
Coefficient of Variation of ROE	34.7%	D-3	D-3

Figure 45 also includes the data exhibits in which the appropriate risk premia for each of the size measures can be found. For example, for use in the Buildup 3 method, risk premia over the risk-free rate (RP_{m+u}) for "coefficient of variation of operating margin" are found in Exhibit D-2. For use in the Buildup 3-Unlevered method, unlevered risk premia over the risk-free rate ($RP_{m+u, unlevered}$) for "coefficient of variation of operating margin" are also found in Exhibit D-2.

In each of the following examples of using the *Risk Study* to estimate COE, the subject company risk measures summarized in Figure 45 will be used (average operating margin of 14.6 percent, for instance, will be used in all examples).

Estimating Cost of Equity Capital Using the "Buildup 3" Method
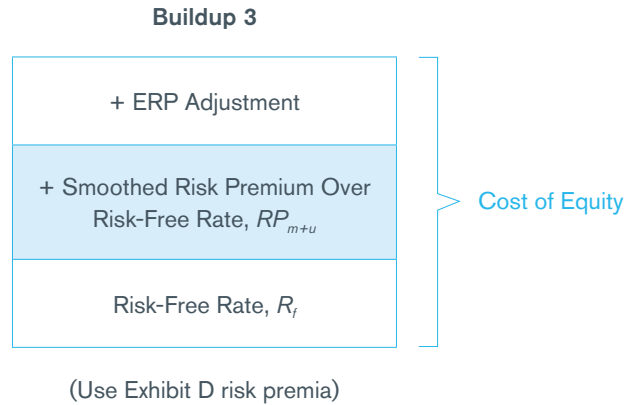
Buildup 3

(Use Exhibit D risk premia)

The buildup method is an additive model commonly used for calculating the required rate of return on equity. As the name implies, successive "building blocks" are summed, each representing the additional risk inherent to investing in alternative assets. An example of this is the extra return (i.e. "premium"), that investors demand for investing in stocks versus investing in a riskless security.[173]

Risk Premia Over the Risk-Free Rate, RP_{m+u}

The risk premia developed in the *Risk Study* (RP_{m+u}) take the form of "risk premia over the risk-free rate", but are slightly different from the risk premia over the risk-free rate (RP_{m+s}) that are developed in the *Size Study*, which are a measure of risk in terms of the combined effect of *market risk* and *size risk*.[174] Because operating efficiencies (or lack thereof) of the subject company are being captured by the use of accounting-based risk measures, the difference in the average rate of return for each risk-based portfolio over the sample period and the income return earned of long-term Treasury bonds (using *SBBI* data) is a measure of risk in terms of the combined effect of *market risk*, and *company-specific risk* (RP_{m+u}).[175] The result is a clear direct relationship between fundamental risk and premium over long-term bond yields. As fundamental risk increases, the return over the risk-free rate (i.e. "excess return") tends to increase.

The RP_{m+u} risk premia can be added to the risk-free rate (R_f) to estimate cost of equity capital using the Buildup 3 method.

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[172] Coefficient of variation is defined here as the standard deviation divided by the mean. For example (using a Microsoft Excel formula), the coefficient of variation of operating margin of the hypothetical subject company used in all examples = *STDEV(16.7,15.0,15.3,10.7,15.6)/AVERAGE(16.7,15.0,15.3,10.7,15.6)*.

[173] Throughout the *Report* the risk-free asset is represented by the yield on a 20-year constant maturity Treasury bond.

[174] For a detailed discussion of how premia over the risk-free rate are calculated, see "The Difference Between 'Risk Premia Over the Risk-Free Rate' and 'Risk Premia Over CAPM'" on page 43.

[175] Because these premia have an embedded measure of market (i.e. "beta") risk, these premia are appropriate for use in "buildup" methods that do not otherwise include a measure of market risk, but *are not* appropriate for use in models (e.g. CAPM) that *already have* a measure of market (beta) risk. *Risk Study* risk premia over the risk-free rate (RP_{m+u}) are published in Exhibits D-1, D-2, and D-3 of the *Risk Premium Report*.

The Risk Study

The "Buildup 3" Equation

As an alternative to the basic buildup equation (see page 50), one can use the *Risk Study* to develop a risk premium for the subject company for which RP_m (the market premium) and RP_u (the company-specific risk premium) are combined into a single premium, RP_{m+u}. The basic buildup equation therefore becomes:

$E(R_i) = R_f + RP_{m+u}$

where:

$E(R_i)$ = Expected rate of return on security i (this is "cost of equity capital", or "COE")

R_f = risk-free rate as of the valuation date (typically a long-term US Treasury bond yield)

RP_{m+u} = risk premium for the subject company for which RP_m (the market premium) and RP_u (the company-specific risk premium) are combined into a single premium.

One final important modification of the basic buildup formula is needed: the Equity Risk Premium (ERP) Adjustment. The equity risk premium adjustment is made to reconcile the historical data presented in the *Risk Premium Report* with the forward-looking ERP chosen by the individual analyst as of valuation date.[176]

The ERP Adjustment is simply the difference between the user's own forward-looking ERP and the historical 1963–2012 ERP (4.5%).[177] For example, many users of the *Report* use the Duff & Phelps Recommended ERP, which is 5.5 percent at the end of 2012).[178, 179] In this case, the ERP Adjustment would be 1.0 percent (5.5%–4.5%).

Adding the ERP Adjustment to the basic buildup formula produces the full equation for the "Buildup 3" method:

$COE_{Buildup\ 3} = R_f + RP_{m+u} + ERP\ Adjustment$

The Buildup 3 method is a straightforward way of estimating cost of equity capital (COE) using the historical "risk premiums over the long-term risk-free rate" (RP_{m+u}) presented in Exhibits D-1 through D-3. It is important to understand that because the risk premia presented in the D exhibits have an embedded measure of market (i.e. "*beta*") risk, they are appropriate only for use in "buildup" methods that do not otherwise include a measure of market risk; these premia are *not* appropriate for use in models (e.g. CAPM) that already have a measure of market (beta) risk.[180]

As noted previously, the *2013 Risk Premium Report* provides two ways for analysts to match their subject company's size (or risk) characteristics with the appropriate smoothed premia from the data exhibits: the "guideline portfolio" method and the "regression equation" method.[181] In general, the regression equation method is preferred because this method allows for interpolation between the individual guideline portfolios, although the guideline portfolio method is less complicated, and more direct. Examples of both the guideline portfolio method and the regression equation method follow, starting with the simpler guideline portfolio method.

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[176] The "ERP Adjustment" is necessary in the *Size Study*'s "Buildup 1" method and "Buildup 1-Unlevered" method, and in the *Risk Study*'s "Buildup 3" method and "Buildup 3-Unlevered" method. See page 12 for more a detailed discussion of the equity risk premium adjustment.

[177] Calculated as the annual S&P 500 Index return minus the average annual long-term *SBBI* government bond income return over the time horizon 1963–2012. Source: Morningstar *EnCorr* software.

[178] For more information on the equity risk premium, see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2011), Chapter 9, "Equity Risk Premium", pages 115–158.

[179] See Roger J. Grabowski, "Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of 'Flight to Quality'", August 2011, by Roger J. Grabowski. A free copy of this paper is available at www.DuffandPhelps.com/CostofCapital

[180] Please refer to page 68 for examples illustrating how to use size premia in conjunction with CAPM to estimate COE.

[181] See page 19 for a detailed explanation of the differences between the guideline portfolio method and the regression equation method.

The Risk Study

Example 5a: Buildup 3 Method (using guideline portfolios)

Three pieces of information are needed to estimate the cost of equity capital using the Buildup 3 method using "guideline portfolios": a risk-free rate (R_f), a risk premium over the risk-free rate (RP_{m+u}), and an ERP Adjustment (if necessary). All of the information needed is summarized in Figure 46.

Figure 46: Information Needed to Estimate COE Using Buildup 3 and Guideline Portfolios
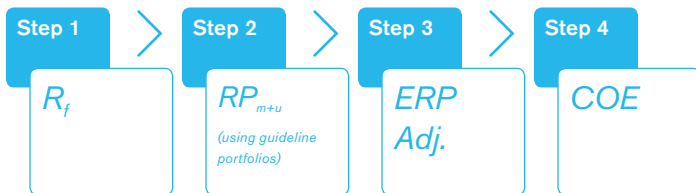
This example utilizes the long-term risk-free rate (R_f) and the ERP Adjustment established in a previous example (the *Size Study*'s Buildup 1 method using "guideline portfolios"; see page 51). This mirrors the fact that for any given valuation engagement the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may be (and probably will be) different than the ones used in the examples. The only missing ingredients needed to estimate COE are the risk premia over the risk-free rate (RP_{m+u}), as summarized in Figure 47.

Step 1, Risk-Free Rate (R_f): The risk-free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the normalized long-term treasury yield of 4.0 percent established in Example 1a (on page 51).

Step 2, Risk Premium Over Risk-Free Rate (RP_{m+u}): Match the various fundamental risk measures of the subject company with the guideline portfolios composed of companies of similar fundamental risk measures in Exhibits D-1 through D-3, and identify the corresponding smoothed average risk premium.

The subject company in this example has an average operating margin of 14.6 percent, and the appropriate data exhibit is Exhibit D-1 (see Figure 45 on page 87).

An abbreviated version of Exhibit D-1 is shown in Figure 48 (on next page). Of the 25 portfolios, the portfolio that has an average operating margin *closest* to the subject company's 14.6 percent is Portfolio 9 (14.62%). The corresponding smoothed average risk premium (RP_{m+u}) is 8.48 percent (8.5%, rounded).

Figure 47: Needed–Smoothed Risk Premia Over the Risk-Free Rate (RP_{m+u}) Using Guideline Portfolios

	Risk Measure	Appropriate Exhibit	Guideline Portfolio	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+u}		Step 3 ERP Adjustment		Step 4 COE
Operating Margin	14.6%	D-1	?	4.0%	+	?	+	1.0%	=	
Coefficient of Variation of Operating Margin	15.8%	D-2	?	4.0%	+	?	+	1.0%	=	
Coefficient of Variation of ROE	34.7%	D-3	?	4.0%	+	?	+	1.0%	=	
		Mean (average) values		4.0%	+		+	1.0%	=	
		Median (typical) values		4.0%	+		+	1.0%	=	

The Risk Study

Match all of the subject company's risk measures in this fashion. For example, the subject company in this example has a "coefficient of variation of operating margin" of 15.8 percent. Of the 25 guideline portfolios in Exhibit D-2 (not shown here), the portfolio that has a coefficient of variation of operating margin *closest* to the subject company's 15.8 percent coefficient of variation of operating margin is Portfolio 14 (15.4%). The corresponding smoothed average risk premium is 9.1 percent. In the case of the third risk measure, the subject company has a "coefficient of variation of ROE" of 34.7 percent. Of the 25 guideline portfolios in Exhibit D-3 (not shown here), the portfolio that has a coefficient of variation of ROE *closest* to the subject company's 34.7 percent coefficient of variation of ROE is Portfolio 14 (34.97%). The corresponding smoothed average risk premium is 9.3 percent.

At this point, all of the available risk measures for the subject company have been matched to the closest guideline portfolio in the appropriate exhibit, and the corresponding smoothed average risk premium has been identified for each, and Step 2 is complete.

Step 3, Equity Risk Premium (ERP) Adjustment: The ERP Adjustment is needed to account for any difference in the user's own ERP estimate and the historical (1963–2012) ERP. This example utilizes the ERP Adjustment (1.0%) established in Example 1a (page 51).

Figure 48: Exhibit D-1 (abbreviated)

Companies Ranked by Operating Margin
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Portfolio Rank	Average Operating Margin	Log of Average Op Margin	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Arithmetic Average Return	Arithmetic Average Risk Premium	Arithmetic Average Unlevered Risk Premium	*Smoothed Average Risk Premium*	Average Debt to Market Value of Equity	Average Debt/MVIC
1	37.08%	-0.43	65	0.88	16.68%	12.93%	14.83%	14.83%	14.83%	14.83%	25.01%
2	29.41%	-0.53	52	0.82	17.19%	11.24%	14.83%	14.83%	14.83%	14.83%	27.70%
3	24.38%	-0.61	62	0.85	17.20%	13.11%	14.83%	14.83%	14.83%	14.83%	26.29%
4	22.11%	-0.66	43	0.93	16.70%	12.87%	14.83%	14.83%	14.83%	14.83%	22.92%
5	19.82%	-0.70	52	0.99	18.14%	14.38%	14.83%	14.83%	14.83%	14.83%	19.70%
6	18.17%	-0.74	51	1.05	17.57%	14.02%	14.83%	14.83%	14.83%	14.83%	17.33%
7	16.69%	-0.78	49	1.09	18.88%	14.85%	14.83%	14.83%	14.83%	14.83%	17.77%
8	15.60%	-0.81	42	1.11	19.83%	14.49%	14.83%	14.83%	14.83%	14.83%	18.39%
9	**14.62%**	-0.84	56	1.16	19.88%	16.38%	9.63%	8.73%	*8.48%*	*23.79%*	19.22%
10	13.59%	-0.87	49	1.16	20.57%	15.41%	14.83%	14.83%	14.83%	14.83%	20.04%
24	3.59%	-1.45	48	1.28	26.03%	19.70%	12.95%	11.35%	*13.47%*	*43.60%*	30.36%
25	2.04%	-1.69	100	1.28	28.42%	20.28%	13.53%	11.95%	*15.47%*	*42.76%*	29.95%

Note: Some values intentionally blurred.

The Risk Study

Step 4, Estimate Cost of Equity (COE): With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using the Buildup 3 method (using guideline portfolios) is now completed. The risk premiums over the risk-free rate (RP_{m+u}) can be added to the risk-free rate (R_f) and the ERP Adjustment to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 49.

The range of COE estimates for the hypothetical subject company in this example is 13.5 percent to 14.3 percent, with an average of 14.0 percent, and a median of 14.1 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" estimate in the group.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the risk premium based on size measures, this difference may be an indication of the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.[182]

Figure 49: Buildup 3 COE Inputs (using guideline portfolios)

	Risk Measure	Appropriate Exhibit	Guideline Portfolio	Step 1 Risks-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+u}		Step 3 ERP Adjustment		Step 4 COE*
Operating Margin	14.6%	D-1	9	4.0%	+	8.5%	+	1.0%	=	13.5%
Coefficient of Variation of Operating Margin	15.8%	D-2	14	4.0%	+	9.1%	+	1.0%	=	14.1%
Coefficient of Variation of ROE	34.7%	D-3	14	4.0%	+	9.3%	+	1.0%	=	14.3%
		Mean (average) values		4.0%	+	8.9%	+	1.0%	=	14.0%
		Median (typical) values		4.0%	+	9.1%	+	1.0%	=	14.1%

* Difference(s) due to rounding.
[182] *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Factors typically referred to as "company-specific" risk factors can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are unique to the subject company.

The Risk Study

Example 5b: Buildup 3 Method (using regression equations)

When the subject company risk measures do not exactly match the respective average risk measure of the guideline portfolios, the data exhibits provide a straightforward way to interpolate an "exact" risk premium over the risk-free rate between guideline portfolios using the "regression equation" method.

The *only* difference between estimating cost of equity capital (COE) using the Buildup 3 method using "guideline portfolios" (as in the previous example) and COE using the Buildup 3 method using "regression equations" is how the risk premia over the risk-free rate (RP_{m+u}) are identified in Step 2.

Figure 50: Steps Needed to Estimate COE Using Buildup 3 and Regression Equations

In the previous example, the smoothed average risk premia *published* in the report for the appropriate guideline portfolios were used to estimate cost of equity capital.[183] In this example, however, the regression equations found in each of the data exhibits will be used to calculate "custom" interpolated risk premia, based upon the specific risk measures of the subject company.

Please note that this example utilizes the long-term risk-free rate (R_f) and the ERP Adjustment established in a previous example (the *Size Study*'s Buildup 1 method using "guideline portfolios"; see page 51). This mirrors the fact that for any given valuation engagement the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may be (and probably will be) different than the ones used in the examples. The only missing ingredients needed to estimate COE are the risk premia over the risk-free rate (RP_{m+u}), as summarized in Figure 51.

Figure 51: Buildup 3 COE Inputs (using regression equations)

	Risk Measure	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+u}		Step 3 ERP Adjustment		Step 4 COE
Operating Margin	14.6%	D-1	4.0%	+	?	+	1.0%	=	
Coefficient of Variation of Operating Margin	15.8%	D-2	4.0%	+	?	+	1.0%	=	
Coefficient of Variation of ROE	34.7%	D-3	4.0%	+	?	+	1.0%	=	
		Mean (average) values	4.0%	+		+	1.0%	=	
		Median (typical) values	4.0%	+		+	1.0%	=	

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[183] The smoothed risk premia published in the *Risk Premium Report* are based upon the average size (or risk) measure of each of the respective guideline portfolios.

The Risk Study

Step 1, Risk-Free Rate (R_f): The risk-free rate is typically a long-term US Treasury bond yield as of the valuation date. This example utilizes the normalized long-term treasury yield of 4.0 percent established in Example 1a (page 51).

Step 2, Risk Premium Over the Risk-Free Rate (RP_{m+u}): The hypothetical subject company in this example has an average operating margin of 14.6 percent, and the appropriate data exhibit is Exhibit D-1[184]. In this case one would expect that the smoothed average risk premium over the risk-free rate (RP_{m+u}) would fall somewhere between 8.24 percent (the smoothed risk premium over the risk-free rate for Portfolio 8) and 8.48 percent (the smoothed risk premium over the risk-free rate for Portfolio 9), as illustrated in Figure 52:

An easy way to calculate a custom interpolated risk premium over the risk-free rate (RP_{m+u}) "in between" Portfolio 8 and Portfolio 9 is by using the regression equations provided for this purpose in each of the data exhibits. The regression equations are located in the same spot in each of the data exhibits (see Figure 7 on page 20).[185]

The regression equation provided in Exhibit D-1, which includes 25 portfolios ranked by operating margin[186], is:

*Smoothed Premium = 1.643% − 8.182% * Log (Operating Margin)*

To calculate an interpolated risk premium for the subject company, substitute the subject company's 14.6 percent operating margin into the regression equation as follows[187]:

*Smoothed Premium = 1.643% − 8.182% * Log (14.6%)*

*8.47% (or 8.5% rounded) =1.643% − 8.182% * (-0.84)*

Figure 52: Exhibit D-1 (abbreviated)

Companies Ranked by Operating Margin
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 2012

Note: Some values intentionally blurred.

Portfolio Rank	Average Operating Margin	Log of Average Op Margin	Number as of 2012	Beta (SumBeta) Since '63	Standard Deviation of Returns	Arithmetic Average Return	Arithmetic Average Risk Premium	Arithmetic Average Unlevered Risk Premium	Smoothed Average Risk Premium	Average Debt to Market Value of Equity	Average Debt/MVIC
1	37.08%	-0.43	65	0.88	16.68%	12.93%	14.83%	14.83%	14.83%	14.83%	25.01%
2	29.41%	-0.53	52	0.82	17.19%	11.24%	14.83%	14.83%	14.83%	14.83%	27.70%
3	24.38%	-0.61	62	0.85	17.20%	13.11%	14.83%	14.83%	14.83%	14.83%	26.29%
4	22.11%	-0.66	43	0.93	16.70%	12.87%	14.83%	14.83%	14.83%	14.83%	22.92%
5	19.82%	-0.70	52	0.99	18.14%	14.38%	14.83%	14.83%	14.83%	14.83%	19.70%
6	18.17%	-0.74	51	1.05	17.57%	14.02%	14.83%	14.83%	14.83%	14.83%	17.33%
7	16.69%	-0.78	49	1.09	18.88%	14.85%	14.83%	14.83%	14.83%	14.83%	17.77%
8	15.60%	-0.81	42	1.11	19.83%	14.49%	7.74%	6.91%	8.24%	22.53%	18.39%
Subject Company	14.64%								?		
9	14.62%	-0.84	56	1.16	19.88%	16.38%	9.63%	8.73%	8.48%	23.79%	19.22%
10	13.59%	-0.87	49	1.16	20.57%	15.41%	14.83%	14.83%	14.83%	14.83%	20.04%
24	3.59%	-1.45	48	1.28	26.03%	19.70%	14.83%	14.83%	14.83%	14.83%	30.36%
25	2.04%	-1.69	100	1.28	28.42%	20.28%	14.83%	14.83%	14.83%	14.83%	29.95%

[184] The same three risk measures (for a hypothetical subject company) are used in all examples of estimating COE using the *Risk Study*, as outlined in Figure 45 on page 87.

[185] In addition to regression equations for interpolating risk premia between guideline portfolios in the *Risk Study*'s D exhibits, the *Size Study*'s A and B exhibits also provide regression equations for easy interpolation of risk premia between guideline portfolios, as do the C exhibits (for unlevered "A" exhibit risk premia).

[186] Please note that each exhibit has a *different* regression equation.

[187] The logarithmic relationship is base-10, and the risk data is in percent, such that the log of 10 percent is log (10%), and not log (10). The formula to calculate a value's base-10 logarithm in Microsoft Excel is "=log (value)". Also note that the "*" used in the regression equation is the symbol used in Microsoft Excel to denote the multiplication symbol, "x". The "*" format is used to denote multiplication in the regression equations in the data exhibits.

The Risk Study

Interpolate smoothed risk premium for each fundamental risk measure available for the subject company using the regression equations from the data exhibits. For example, the subject company in this example has a "coefficient of variation of operating margin" of 15.8 percent. The regression equation provided in Exhibit D-2 is:

Smoothed Premium = 12.749% + 4.487% * Log (CV Op. Margin)

The interpolated smoothed risk premium is therefore 9.1 percent (12.749% + 4.487% * (-0.80)).

In the case of the third risk measure, the subject company has a "coefficient of variation of ROE" of 34.7 percent. The regression equation provided in Exhibit D-3 is:

Smoothed Premium = 10.237% + 2.137% * Log (CV ROE)

The interpolated smoothed risk premium is therefore 9.3 percent (10.237% + 2.137% * (-0.46)).

After interpolating smoothed risk premia (RP_{m+u}) for the subject company's available risk measures, Step 2 is complete.

Step 3, Equity Risk Premium (ERP) Adjustment: The ERP Adjustment is needed to account for any difference in the analyst's own ERP estimate and the historical (1963–2012) ERP. This example utilizes the ERP Adjustment (1.0%) established in Example 1a (page 51).

Step 4, Estimate Cost of Equity (COE): With the completion of Steps 1 through 3, the information needed to estimate a base cost of equity capital using the Buildup 3 method (using regression equations) is now completed. The risk premiums over the risk-free rate (RP_{m+u}) can be added to the risk-free rate (R_f) and the ERP Adjustment to estimate an indicated cost of equity capital (COE) for the subject company, as illustrated in Figure 53.

The range of COE estimates for the hypothetical subject company in this example is 13.5 percent to 14.3 percent, with an average of 14.0 percent, and a median of 14.2 percent. The mean represents the average estimate, but the mean can be unduly influenced by very large or very small "outliers". For this reason, the median estimate is generally preferred to the mean. The median estimate tends to not be as heavily influenced by very large or very small outliers, and can be considered a measure of the "typical" estimate in the group.

Use of fundamental accounting measures of risk allows for direct assessment of the riskiness of the subject company. For example, if the appropriate risk premium for the subject company when measuring risk by one or more fundamental risk measures is different than the risk premium based on size measures, this difference may be an indication of the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.[188]

Figure 53: Buildup 3 COE Inputs (using regression equations)

	Risk Measure	Appropriate Exhibit	Step 1 Risk-Free Rate, R_f		Step 2 Smoothed Premium Over Risk-Free Rate, RP_{m+u}		Step 3 ERP Adjustment		Step 4 COE*
Operating Margin	14.6%	D-1	4.0%	+	8.5%	+	1.0%	=	**13.5%**
Coefficient of Variation of Operating Margin	15.8%	D-2	4.0%	+	9.1%	+	1.0%	=	14.2%
Coefficient of Variation of ROE	34.7%	D-3	4.0%	+	9.3%	+	1.0%	=	**14.3%**
		Mean (average) values	4.0%	+	9.0%	+	1.0%	=	**14.0%**
		Median (typical) values	4.0%	+	9.1%	+	1.0%	=	**14.2%**

---

*   Differences due to rounding.

[188] *Valuing a Business*, 4th ed., Pratt et al, McGraw-Hill (2000), p 181. Factors typically referred to as "company-specific" risk factors can include concentration of customer base, key person dependence, key supplier dependence, or any number of other factors that are unique to the subject company.

The High-Financial-Risk Study

The information and data in the *Risk Premium Report* and in the online *Risk Premium Calculator*[194] is primarily designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy, and for which a "going concern" assumption is appropriate. A set of "high-financial-risk" companies is set aside and analyzed separately in the *High-Financial-Risk Study*.

The companies analyzed in the *High-Financial-Risk Study* are identified in a two-step process. First, companies that are losing money, have high leverage, or are in bankruptcy are identified and eliminated from the base set of companies used in the *Size Study* and *Risk Study*.[195, 196] It is possible to imagine companies that don't have any of these characteristics but could still be classified as high-financial-risk (i.e. "distressed"), and it is also possible to imagine companies which do have one or more of these characteristics but are *not* distressed.

For this reason, these companies are further scrutinized in a second test where they are ranked by the appropriate Altman z-Score (for "manufacturing" companies or for "service" companies).[197, 198] Those companies identified as being in the "safe zone" (as defined by their z-Score) failed the first test, but passed the second test (z-Score), and are set aside and not used in any further analysis due to the inconclusive results. The remaining companies failed *both* the first test and the second test, and are placed in either the "gray" or "distressed" zone (as defined by their z-Score). The resulting base set of high-financial-risk companies is composed largely of companies whose financial condition is significantly inferior to the average, financially "healthy" public company.

The results of the *High-Financial-Risk Study* are presented in the H exhibits. The H exhibits provide risk premia that may be used in both buildup and CAPM estimates of cost of equity capital if the individual analyst has determined that the subject company is "high-financial-risk".[199]

In cases in which the individual analyst has determined that the subject company is "high-financial-risk", the high-financial-risk premia reported in the H exhibits should be used *instead of* the returns reported in the *Size Study*, and *not* added to those returns.

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[194] The online *Duff & Phelps Risk Premium Calculator* automatically calculates, properly applies, and fully documents the ERP Adjustment. The *Risk Premium Calculator* is available through Business Valuation Resources (BVR) and ValuSource.

[195] For a detailed discussion of how the high-financial-risk portfolios are created, see "High-Financial-Risk Study" in the portfolio methodology section on page 7.

[196] The number of companies eliminated in this screen varies from year to year. These companies represented up to 25% of the data set in recent years, but less than 5% in 1963. Certain technical changes in methodology have resulted in a greater number of companies falling into the high-financial-risk database than in versions of this study published prior to 2000.

[197] Altman z-Score is an accounting-data-based method designed to assess financial condition and developed originally for assessing the likelihood of bankruptcy. E. I. Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *The Journal of Finance*, Vol. 23, No. 4 (Sep., 1968), pp. 589-609; "Predicting Financial Distress of Companies: Revisiting the s-Score and Zeta Models," July 2000; "Revisiting Credit Scoring Models in a Basel 2 Environment," May 2002.

[198] Service industry companies are those SIC codes: 7200, 7300, 7500, 7600, 8000, 8100, 8200, 8300, 8400, 8700. Manufacturing are all other SIC codes, with the exception of SICs beginning with "6" (financial institutions) or "9" (government). SIC 6 and SIC 9 are not included in the *Report*'s analysis.

[199] The decision to apply a high-financial-risk premium is ultimately dependent on the analyst's professional judgment, based upon the analyst's detailed knowledge of the subject company.

The High-Financial-Risk Study

The High-Financial-Risk "H" Exhibits

Exhibit H-A is the high-financial-risk equivalent of the A exhibits. "High-financial-risk premia over the risk-free rate" for use in a buildup method are found in the H-A exhibits. These premia can be added to the risk-free rate to estimate the cost of equity capital for a company that has been judged by the analyst to be high-financial-risk.

Exhibit H-B is the high-financial-risk equivalent of the B exhibits. "High-financial-risk premia over CAPM" (i.e. "size premia") for use with the CAPM method are found in the H-B exhibits. These premia can be used in the CAPM to estimate the cost of equity capital for a company that has been judged by the analyst to be high-financial-risk.

Exhibit H-C is the high-financial-risk equivalent of the C exhibits. The H-C exhibits can be used to compare the subject company's fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of companies with similar z-Scores.

Figure 58: The A, B, and C Exhibits and Corresponding High-Financial-Risk Exhibits
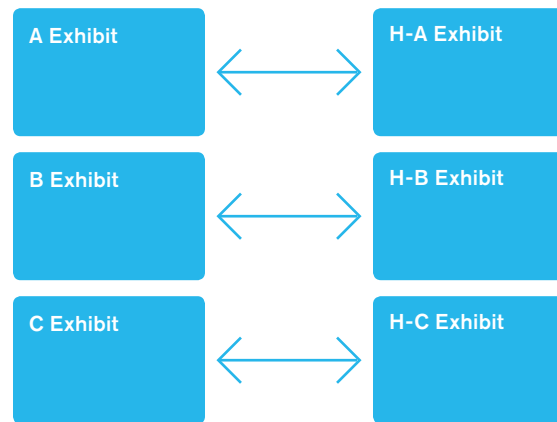
Why isn't there an H-D exhibit? In the *Risk Study*'s *D* exhibits, in addition to operating margin, two other measures of risk are examined (coefficient of variation in operating margin and coefficient of variation in return on equity). Because the denominators of these ratios are often negative for companies in the high-financial-risk portfolio as a result of either negative earnings or negative book value of equity, developing comparable "high-financial-risk" premia for these frequently results in meaningless statistics.

The High-Financial-Risk Study

Altman z-Score

Altman's z-Score was originally designed as a measure to predict the risk of failure up to two years prior to distress for a sample of manufacturing companies using financial data prepared according to the standards of the day. The accuracy of predicting the risk of failure diminished substantially as the lead time increased. The z-Score resulted from a statistical analysis of company data using the statistical technique of multiple discriminant analysis.

Altman has since offered improvements on the original z-Score, but the original z-Score is still frequently calculated as a convenient metric that captures within a single statistic a number of disparate financial ratios measuring liquidity, profitability, leverage, and asset turnover.[200]

Z-Score ratios are not strictly comparable across industries or across time (for instance, one would expect large differences in asset turnover among an industrial company or a retailer), and as such, are not used here as a predictor of bankruptcy per se, but as mechanism for ranking the high-financial-risk companies by their relative levels of distress.

The following z-Score model for publicly-traded "manufacturing" companies (i.e. excluding service industry companies) is used in preparing the analyses presented in the H-A, H-B, and H-C exhibits:

$$z = 1.2\ x_1 + 1.4\ x_2 + 3.3\ x_3 + 0.6\ x_4 + 0.999\ x_5$$

where:

z = Overall index

x_1 = Net working capital / total assets

x_2 = Retained earnings / total assets

x_3 = Earnings before interest and income taxes / total assets

x_4 = *Market* value of common equity / book value of total liabilities

x_5 = Sales / total assets

The companies are then sorted by z-Score into three portfolios:

- $z > 2.99$ = "safe zone"
- $1.80 < z < 2.99$ = "gray zone"
- $z < 1.80$ = "distress zone"

Companies in the "safe" zone (z-Score greater than 2.99) are set aside and not used in any further analysis. Companies in the "gray" zone (z-Score between 1.80 and 2.99) and companies in the "distressed" zone (z-Score less than 1.80) are used to form the portfolios from which the statistics presented in H-A, H-B, and H-C exhibits are calculated. Portfolios are rebalanced annually (i.e. the companies are re-ranked and sorted at the beginning of each year). Portfolio rates of return were calculated using an equal-weighted average of the companies in the portfolio.

[200] In applying any of the z-Score equations cited here, express the ratios in terms of their decimal equivalents (e.g., x_1 = working capital / total assets = 0.083).

The High-Financial-Risk Study

The following z"-Score model for publicly-traded "service" industry high-financial-risk companies is used in preparing the analyses presented in the H-A, H-B, and H-C exhibits:

$z'' = 6.56 \, x_1 + 3.26 \, x_2 + 6.72 \, x_3 + 1.05 \, x_4$

where:

z'' = Overall index

x_1 = Net working capital / total assets

x_2 = Retained earnings / total assets

x_3 = Earnings before interest and income taxes / total assets

x_4 = *Book* value of common equity / book value of total liabilities

The companies are then sorted by z"-Score into three portfolios.

- $z'' > 2.60$ = "safe zone"
- $1.10 < z'' < 2.60$ = "gray zone"
- $z'' < 1.10$ = "distress zone"

Companies in the "safe" zone (z"-Score greater than 2.60) are set aside and not used in any further analysis. Companies in the "gray" zone (z"-Score between 1.10 and 2.59) and companies in the "distressed" zone (z"-Score less than 1.10) are used to form the portfolios from which the statistics presented in H-A, H-B, and H-C exhibits are calculated. Portfolios are rebalanced annually (i.e. the companies are re-ranked and sorted at the beginning of each year). Portfolio rates of return were calculated using an equal-weighted average of the companies in the portfolio.

Again, in both cases (manufacturing and service), we are not using the z-Score or z"-Score as a predictor of bankruptcy. Rather, companies are ranked in the *High-Financial-Risk Study* based on their relative levels of distress, using z-Score and z"-Score as proxies for "distress".

Non-Public Companies and z'-Score

The traditional z-Score was developed using data for publicly traded companies, and one of the statistics utilizes stock price. This creates problems for application of the data to non-public companies. Altman developed a similar model using only the financial statement data for non-public companies. If the subject company is not publicly traded and not in the service industry, then the analyst can calculate a z-Score for non-public companies (the z'-Score) to compare with the data in the accompanying exhibits:

$z' = 0.717 \, x_1 + 0.847 \, x_2 + 3.107 \, x_3 + 0.420 \, x_4 + 0.998 \, x_5$

where:

z' = Overall index

x_1 = Working capital / total assets

x_2 = Retained earnings / total assets

x_3 = Earnings before interest and income taxes / total assets

x_4 = *Book* value of common equity / book value of total liabilities

x_5 = Sales / total assets

The z'-Score's "zones of discrimination" loosely approximate the boundaries used to seperate the z-Score and z"-Score ranked companies into portfolios, and are as follows:

- $z' > 2.90$ = "safe zone"
- $1.23 < z' < 2.90$ = "gray zone"
- $z' < 1.23$ = "distress zone"

While the H-A, H-B, and H-C exhibits are sorted by using the publically-traded company equations (z-Score for manufacturing companies and z"-Score for service companies) and are not strictly comparable to the z'-Score for non-public companies, the returns reported in these exhibits can be useful in developing cost of equity estimates based on the relative zones of discrimination.

The High-Financial-Risk Study

Measurement of Historical Risk Premiums

The high-financial-risk Study's H exhibits report average historical risk premiums for the period 1963 (the year that the *Compustat* database was inaugurated) through 2012. A long-run average historical risk premium is often used as an indicator of the expected risk premium of a typical equity investor. Returns are based on dividend income plus capital appreciation and represents returns after corporate taxes (but before owner level taxes).

To estimate historical risk premiums, an average rate of return is first calculated for each portfolio over the sample period. Portfolios with fewer than six companies in any given year are excluded in the averages. Lastly, the average income return earned on long-term Treasury bonds is subtracted from the portfolios' returns over the same period (using *SBBI* data) to arrive at an average historical risk premium for investments in equity.

The "H" Exhibits – Summary of Data Presented

Each of the exhibits H-A, H-B, and H-C displays one line of data for each of the the z-Score- and z"-Score-ranked portfolios. These exhibits include the statistics outlined in Table 9.

For comparative purposes, the average returns from the *SBBI* series for large companies (essentially the S&P 500 Index), small companies, and long-term government bond income returns for the period 1963 through the latest year are also reported on each exhibit.[201]

Table 9: Statistics Reported for the z-Score- and z"-Score-ranked High-Financial-Risk Study's H-A, H-B, and H-C Exhibits

Exhibit H-A	Exhibit H-B	Exhibit H-C
Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).	Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).	Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ($RP_{m+s, high-financial-risk}$).
Standard deviation of annual historical equity returns.	Arithmetic average historical equity return since 1963.	Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.
Geometric average historical equity return since 1963.	Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ($RP_{m+s, high-financial-risk}$).	Average debt to market value of equity.
Arithmetic average historical equity return since 1963.	Indicated CAPM premium, calculated as the beta of the portfolio multiplied by the average historical market risk premium since 1963 (measured as the difference between *SBBI* Large Stock total returns and *SBBI* income returns on long-term Treasury bonds).	Beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).
Arithmetic average historical risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963 ($RP_{m+s, high-financial-risk}$).	Premium over CAPM, calculated by subtracting the "Indicated CAPM Premium" from the "Arithmetic Risk Premium" ($RP_{s, high-financial-risk}$).	Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.
Average carrying value of preferred stock plus long-term debt (including current portion) plus notes payable ("Debt") as a percent of MVIC since 1963.		

The High-Financial-Risk Study

The Buildup 1-High-Financial-Risk method for estimating the cost of equity capital using the *Report's* High-Financial-Risk Study is shown in this excerpted version of the *Report*.

Examples for each of the methods of cost of equity capital estimation methods available using the *Report* (see Table 1) are available in the full version *Report*.

Overview of Methods Used to Estimate Cost of Equity Capital Using the High-Financial-Risk Study

The *High-Financial-Risk Study* provides two methods of estimating COE for a subject company that has been determined to be high-financial-risk: "Buildup 1-High-Financial-Risk" and "CAPM-High-Financial-Risk". These methods are summarized in equation format, and summarized in Figure 59 in graphical "building blocks" format.

1) **Buildup 1-High-Financial-Risk**

   $COE_{Buildup\ 1\text{-}High\text{-}Financial\text{-}Risk}$ = (Risk-Free Rate) + (High Financial Risk Premium Over Risk-Free Rate) + (Equity Risk Premium Adjustment)

   Example 7: page 105

2) **Capital asset pricing model (CAPM)-High-Financial-Risk**

   $COE_{CAPM\text{-}High\text{-}Financial\text{-}Risk}$ = (Risk-Free Rate) + (Beta x Equity Risk Premium) + (High-Financial-Risk Size Premium)

   Example 8: page 108

Figure 59: Two Methods of Estimating Cost of Equity Capital with the *High-Financial-Risk Study*[202]
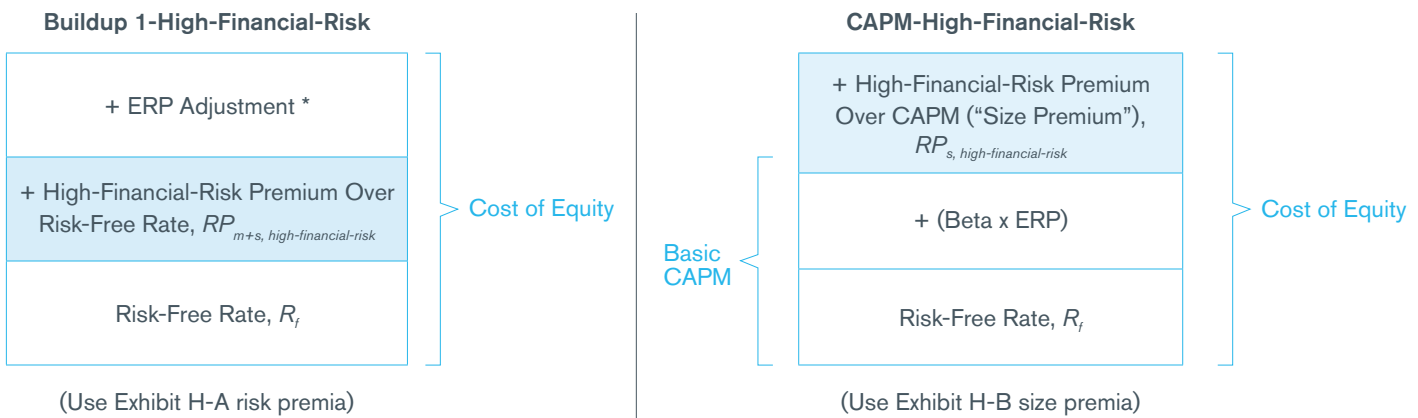
Buildup 1-High-Financial-Risk

+ ERP Adjustment *
+ High-Financial-Risk Premium Over Risk-Free Rate, $RP_{m+s,\ high\text{-}financial\text{-}risk}$
Risk-Free Rate, R_f

> Cost of Equity

(Use Exhibit H-A risk premia)

CAPM-High-Financial-Risk

+ High-Financial-Risk Premium Over CAPM ("Size Premium"), $RP_{s,\ high\text{-}financial\text{-}risk}$
+ (Beta x ERP)
Risk-Free Rate, R_f

> Cost of Equity

Basic CAPM

(Use Exhibit H-B size premia)

*ERP Adjustment: The difference between the historical (1963–2012) equity risk premium (*ERP*) and a user of the *Duff & Phelps Report*'s own forward *ERP* estimate:

ERP Adjustment = User's ERP − Historical (1963–2012) ERP

The ERP Adjustment is made only in the "Buildup 1", "Buildup1-Unlevered", "Buildup 1-High-Financial-Risk", "Buildup 3", and "Buildup 3-Unlevered" methods. Please refer to the individual examples provided for these models for more information.

For a detailed discussion of the ERP Adjustment, see page 12.

[202] The relative sizes of the "building blocks" in Figure 59 do not necessarily represent the relative size of the various inputs. Also note that the names given to the models in the *Risk Premium Report* (e.g. "Buildup 1", "Buildup 2", "Buildup 3", etc.) are naming conventions used within the *Report* to make referring to the different methods easier.

The High-Financial-Risk Study

In this section, the information in Figure 60 will be used to estimate cost of equity capital for a hypothetical non-service (i.e. "manufacturing") subject company.

Figure 60: Subject Company Characteristics (used in all examples)

	(in $millions)		(in $millions)
Market value of equity	$80	Sales	$250
Book value of equity	$100	Current assets	$75
Total assets	$300	Current liabilities	$50
Most recent year EBIT	-$5	Retained earnings	$75

The z-Score equation for a publicly-traded, non-service (i.e. "manufacturing") subject company is:

$$z = 1.2\, x_1 + 1.4\, x_2 + 3.3\, x_3 + 0.6\, x_4 + 0.999\, x_5$$

The inputs (x_1, x_2, x_3, x_4, and x_5) needed for the z-Score equation are calculated as shown in Figure 61:

Substituting these inputs into the z-Score equation yields a z-Score of 1.47:

$$z = 1.2(0.0833) + 1.4(0.2500) + 3.3(-0.0167) + 0.6(0.4000) + 0.999(0.8333)$$

$$1.47 = 0.1000 + 0.3500 + (-0.0550) + 0.2400 + 0.8325$$

Example 7: Estimating Cost of Equity Capital Using the "Buildup 1-High-Financial-Risk" Method

Buildup 1-High-Financial-Risk
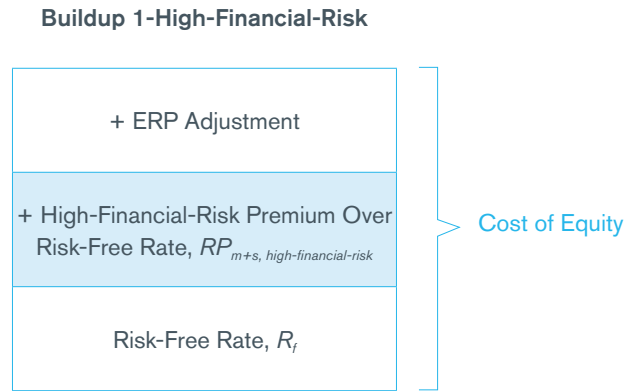
(Use Exhibit H-A risk premia)

The buildup method is an additive model commonly used for calculating the required rate of return on equity. As the name implies, successive "building blocks" are summed, each representing the additional risk inherent to investing in alternative assets. An example of this is the extra return (i.e. "premium"), that investors demand for investing in stocks versus investing in a riskless security.[203, 204]

This example utilizes the long-term risk-free rate (R_f) and the ERP Adjustment established in a previous example (the *Size Study*'s Buildup 1 method using "guideline portfolios"; see page 51). This mirrors the fact that for any given valuation engagement the same risk-free rate and ERP will generally be used in each of the models presented by the individual analyst. Please note that for any given valuation engagement these inputs may (and probably will) be different than the ones used in the examples.

Figure 61: z-Score Inputs Calculation

x_1	=	Net working capital / total assets	= ($75 current assets - $50 current liabilities) / ($300 total assets)	=	0.0833
x_2	=	Retained earnings / total assets	= ($75 retained earnings) / ($300 total assets)	=	0.2500
x_3	=	Earnings before interest and taxes / total assets	= (-$5 EBIT) / ($300 total assets)	=	-0.0167
x_4	=	Market value of common equity / book value of total liabilities	= ($80 market value of equity) / ($300 total assets - $100 book value of equity)	=	0.4000
x_5	=	Sales / total assets	= ($250 sales) / ($300 total assets)	=	0.8333

[203] Throughout this report the risk-free asset (R_f) is represented by the yield on a 20-year constant maturity Treasury Bond.
[204] For a detailed discussion of the buildup model, see "Estimating Cost of Equity Capital Using the 'Buildup 1' Method" on page 49.

The High-Financial-Risk Study

As in the Buildup 1 method, the "Buildup 1-High-Financial-Risk" method requires three pieces of information to estimate the cost of equity capital: a risk-free rate (R_f), a high-financial-risk premium over the risk-free rate ($RP_{m+s, high-financial-risk}$), and an ERP Adjustment (if necessary). All of the information needed is summarized in Figure 62.

Figure 62: Information Needed to Estimate COE Using "Buildup 1-High-Financial-Risk"
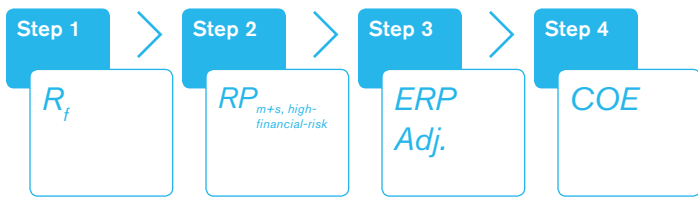
Step 1	Step 2	Step 3	Step 4
R_f	$RP_{m+s, high-financial-risk}$	ERP Adj.	COE

The *only* difference between estimating cost of equity capital (COE) using the Buildup 1 method and estimating COE using the Buildup 1-High-Financial-Risk method is that the "risk premium over the risk-free rate" used in the latter method is a "high-financial-risk premium" ($RP_{m+s, high-financial-risk}$), while the risk premia over the risk-free rate used in the former are not.[205]

Step 1 and Step 3: Because the normalized risk-free rate in Step 1 (4.0%) and the ERP Adjustment in Step 3 (1.0%) established in a previous example are being used in this example[206], the only missing ingredient needed to estimate COE is the high-financial-risk premium over the risk-free rate ($RP_{m+s, high-financial-risk}$):

$COE_{Buildup\ 1-High-Financial-Risk} = R_f + RP_{m+s,high-financial-risk} + ERP\ Adjustment =$

$COE_{Buildup\ 1-High-Financial-Risk} = 4.0\% + RP_{m+s,high-financial-risk} + 1.0\%$

Determination of the high-financial-risk premium in Exhibit H-A for Step 2 is a three-step process (Steps 2a, 2b, and 2c):

Step 2a: Determine whether the characteristics of the subject company better match the characteristics of the companies included in Exhibits A-1 through A-8 (the 25 portfolios) or the characteristics of the high-financial-risk portfolios of companies as described above. The most straightforward way of doing this is to answer the following five questions about the subject company:[207]

- Is the subject company in bankruptcy or in liquidation?

- Is the subject company's "5-year average net income available to common equity" less than zero for the previous five years?

- Is the subject company's "5-year-average operating income" less than zero for the previous five years?

- Has the subject company had a negative book value of equity at any one of the company's previous five fiscal year-ends?

- Does the subject company have a debt-to-total capital ratio of more than 80%?

It is possible to imagine companies that don't have any of these characteristics, but could still be classified as high-financial-risk (i.e. "distressed"), and it is also possible to imagine companies which do have one or more of these characteristics but are *not* distressed.

If you answered "Yes" to one or more of the five questions, it *may* suggest that the subject company's characteristics are more like the companies that make up the "high-financial-risk" portfolios rather than like the "healthy" companies that make up the standard 25 portfolios, *but not necessarily so*. For example, a company may have a debt to total capital ratio greater than 80%, but this does not automatically imply that the company is in distress. A decision by the individual analyst that a company should be treated as "high-financial-risk" should be based on a detailed evaluation of the company's current financial condition and circumstances, and will generally involve more than a review of historical financial statistics and ratios. The decision to apply a high-financial-risk premium is ultimately dependent on the individual analyst's professional judgment and detailed knowledge of the subject company.[208]

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[205] The "risk premia over the risk-free rate" used in the Buildup 1 method are found in the A exhibits. A, B, C, and D risk premia are designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy; the H exhibits are designed to be used to estimate COE for companies that the individual analyst has determined to be "high-financial-risk".

[206] See the *Size Study*'s Buildup 1 method using "guideline portfolios" on page 51.

[207] These five questions mirror the five criteria by which high-financial-risk companies are identified in (and eliminated from) the universe of U.S. companies to form the base set of companies used in the *Size Study* and *Risk Study*.

[208] If the analyst determines that the subject company is not high-financial-risk, the returns reported in the exhibits in the *Risk Premium Report* for the 25 portfolios (the A, B, C, and D exhibits) may be more appropriate for the subject company than the returns reported in the H exhibits.

The High-Financial-
Risk Study

Step 2b: If the individual analyst determines that the subject company's characteristics better match the characteristics of the companies comprising the high-financial-risk portfolios, calculate the z-Score of the subject company using the appropriate z-Score equation:[209]

- z-Score is for publicly-traded, non-service, (i.e. "manufacturing") companies[210]

- z"-Score is for publicly-traded, "service" companies

- z'-Score is non-public, non-service companies.

Step 2c: Lastly, if the z-Score[211] of the subject company indicates that it is in the "gray zone" or "distress zone", match the z-Score of the subject company with the zone composed of companies with similar z-Scores in Exhibits H-A, and identify the corresponding average high-financial-risk premium over the risk-free rate ($RP_{m+s,\ high-financial-risk}$). For this example, the subject company is a manufacturing company with a z-Score of 1.47, placing it in the "distressed" portfolio (z-Scores <1.8; see Figure 63). The corresponding high-financial-risk arithmetic average risk premium is 16.52 percent (16.5% rounded).

Step 4: Estimate a high-financial-risk cost of equity for the subject company by adding the average high-financial-risk premium over the risk-free rate identified in Step 3 ($RP_{m+s,\ high-financial-risk}$) to the risk-free rate R_f and the ERP Adjustment (if appropriate).

$$COE_{Buildup\ 1-High-Financial-Risk} = R_f + RP_{m+s,\ high-financial-risk} + ERP\ Adjustment =$$
$$21.5\% = 4.0\% + 16.5\% + 1.0\%$$

The "high-financial-risk" cost of equity capital estimate for the hypothetical subject company in this example is 21.5 percent.

Figure 63: "Buildup 1-High-Financial-Risk" COE Input
Exhibit H-A, High-Financial-Risk Premia Over the Risk-Free Rate
Companies Ranked by z-Score
Historical Equity Risk Premium: Average Since 1963
High-Financial-Risk Company Data for Year Ending December 31, 2012

Portfolio Rank	Beta (SumBeta) Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Average Risk Premium	Average Debt/MVIC
Manufacturing (z-Score)						
1.8 to 2.99 (gray zone)	1.57	37.22%	14.73%	21.08%	*(blurred)*	46.26%
< 1.8 (distress zone)	1.66	39.58%	16.00%	23.27%	**16.52%**	57.76%
Service Industry (z"-Score)						
1.1 to 2.59 (gray zone)	1.59	42.64%	13.92%	26.97%	*(blurred)*	41.45%
< 1.1 (distress zone)	1.72	46.18%	19.62%	34.45%	*(blurred)*	49.98%

Note: Some values intentionally blurred.

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[209] In all examples here, the z-Score for publicly-traded, non-service (i.e. "manufacturing") companies is used.

[210] While the H-A, H-B, and H-C exhibits are ranked by z-Score and z"-Score and are not strictly comparable to the z'-Score for non-public companies, the returns reported in these exhibits can be useful in developing cost of equity estimates based on the relative zones of discrimination.

[211] Or, as appropriate, z''-Score or z'-Score.

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

A differentiating capability of the *Duff & Phelps Risk Premium Report* is that it includes information about the characteristics of the companies that make up the portfolios that are used to calculate the risk premia and size premia published in the *Report*. This is an important capability because it enables *Report* users to potentially further refine their cost of equity estimate (COE) by gauging how "alike or different" the subject company is compared to the companies that make up the *Report*'s guideline portfolios.

The *Risk Premium Report's* "C" exhibits can be used to gauge whether an increase or decrease adjustment to a risk premium or size premium (and thus, COE) is indicated, based upon the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived.

Figure 66: The C Exhibits providea "link" between the Size Study portfolios and accounting-cased fundamental risk characteristics
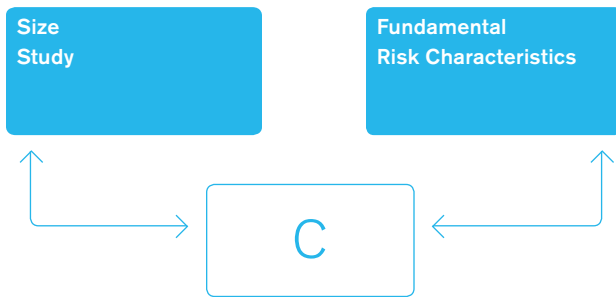
Valuation is an Inherently Comparative Process

Just about any analysis boils down to trying to comparing one thing to another. For example, when "analyzing" the merits of a house we are thinking about purchasing, it's common to compare it to other houses with similar characteristics. While houses that are exactly the same may be available in certain instances, typically what we end up with is a "peer group" of comparable houses that may be similar in many respects, but may still have some differences. If the house we are looking at is the only one in the neighborhood without a swimming pool, we could probably make a pretty good argument that a downward adjustment in price is justified. On the other hand, if the house we are looking at has a two-car garage while all the other houses in the neighborhood have one-car garages, an upward adjustment in price may be unavoidable.

Just as we oftentimes make decisions based upon the alikeness (or difference) between alternatives, the use of a portfolio's average historical rate of return to estimate a discount rate for a subject company is *also* based upon the implicit assumption that the risks of the subject company are quantitatively similar to the risks of the average company in the portfolio. If the risks of the subject company differ materially from the average company in the portfolio, then the estimated discount rate may be less than (or greater than) the discount rate derived using the risk premium or size premium associated with the given portfolio.

"Company-Specific" Risk

A few users of the *Report* have pointed out that using the term "company-specific" in this context might be confusing to some readers, because another use of the term "company-specific" implies risks that in the theoretical sense can be diversified away.[218] Having said that, the intended meaning of the term "company-specific risk" can vary from person to person. For example, is a "company-specific" risk adjustment necessary in a hypothetical case in which the comparison peer group and the subject company are identical in every way? Many analysts would contend that the answer to this question is "no" – although this answer probably has very little to do with the theoretical definition of company-specific risk. What is probably intended is that no further adjustment may be necessary because the peer group in this hypothetical case (being identical to the subject) acts as a "perfect" proxy.

We see valuators regularly make adjustments to COE estimates made under the heading "company-specific" risk, including (but not limited to):

- Adjustments to COE estimates derived from a sample of guideline public companies to account for a subject company having risk characteristics that differ from the peer group.

- Adjustments to COE estimates to account for biased cash flow projections provided to the valuator.

- Adjustments to COE estimates to account for risks accepted by investors that may not hold diversified portfolios of investments.

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[218] COE models generally assume that risks that can be "diversified away" are not compensable; this risk is properly called "unsystematic risk".

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

The third case (the diversified versus undiversified investor) likely comes up often with valuators, since in many cases the owner of the asset being valued may be an investor who is otherwise "undiversified". In these cases, some may conclude that the COE is (at least in part) a function of the investor, and others may even further conclude that the characteristics of the *investor* are paramount, and that the characteristics of the *investment* may be a distant second. These conclusions are quite problematic. An individual investor can indeed have his or her own personal required return, but this may have little to do with the characteristics of the investment compared to the next best opportunity.

Are there businesses that are typically owned by investors that have everything tied up in it? Yes. Are there businesses that are so identified with or dependent on their owner in some fashion that one might surmise that there is only one "natural owner" of the firm? Yes. But, this is ultimately a characteristic of the investment, and not the investor. It is at least plausible in these cases that it is not so much that the owner is "undiversified", but that the investment is "undiversifiable", so to speak. The proper comparison may be to other investments with similar characteristics, and *not* a comparison of the investor to other investors of similar characteristics.

Using the "C" Exhibits to Refine Cost of Capital Estimates

The *Risk Premium Report* is designed to assist the analyst in estimating the cost of equity capital for the subject company as if it were publicly traded. That is, the returns reflect the risks and the liquidity of publicly-traded stocks. However, discounting expected net cash flows for a closely held business using an "as if public" cost of capital may not be an accurate estimate of value to the extent that market participants consider other risks associated with investments in closely held businesses. In other words, when estimating the cost of equity capital for a subject company, the risks of the subject company more than likely differ in some respects from the risks of the sample of guideline public companies it is being compared to (i.e., the "peer group").

When we use the *Risk Premium Report*'s risk premia over the risk-free rate from the "A" exhibits or the size premia from the "B" exhibits, the "peer group" is the guideline portfolio in which the subject company falls. Remember that the cost of equity capital estimates developed using the *Report* are still "as if public", even *after* using the C exhibits to gauge the company-specific differences of the subject company and the portfolio(s) to which the subject company is being compared. However, this refined estimate likely better reflects the risk of the subject company as if the stock of the company was publically traded, and had been discounted by the market's assessment of its company-specific risk characteristics (as measured by its accounting-based fundamental risk measures).

The "C" exhibits provide information about the companies that comprise the 25 portfolios that are used to create the various risk premia and size premia published in the *Report*. This information can be used to gauge how "alike or different" the subject company is compared to the average company in these portfolios, making it possible for *Report* users to further refine their COE estimate.

The "C" exhibits provide the following three comparative risk characteristics (i.e. "accounting-based fundamental risk measures") for each of the 25 portfolios and for each of the 8 size measures of size, each of which can be useful in assessing how "alike or different" the subject company is to the companies that make up the respective guideline portfolio:

- Average operating margin
- Average coefficient of variation of operating margin
- Average coefficient of variation of ROE

To calculate the statistics included in Exhibits C-1 through C-8, the fundamental risk characteristics are calculated for the same size-ranked portfolios that are created in the *Size Study*. For example, Exhibit A-1 is comprised of 25 portfolios ranked by market value of equity. To calculate the fundamental risk characteristics found in Exhibit C-1, the three fundamental risk measures used to rank the portfolios in the *Risk Study* (five-year operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity) are calculated for each of the 25 (market-value-of-equity-ranked) portfolios in Exhibit A-1.

These calculations are then made in the same fashion for each of the 25 size-ranked portfolios created for Exhibits A-2 through A-8 (e.g. for each of the 25 portfolios ranked by "book value of equity" in Exhibit A-2, the three fundamental risk measures are calculated; then for each of the 25 portfolios ranked by "5-year average net income" in Exhibit A-3, the three fundamental risk measures are calculated, etc.).

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

The "C" Exhibits – Summary of Data Presented

In addition to information repeated from the A exhibits, the C exhibits report the additional datapoints for each of the 25 portfolios described in Table 11.

Exhibits C-1 through C-8 also provide unlevered versions of the risk premia over the risk-free rate found in the A exhibits (RP_{m+s}). These unlevered premia ($RP_{m+s,\ unlevered}$) are used in Examples 2a and 2b (see page 60 and 64, respectively) to estimate cost of equity capital assuming a firm is financed 100% with equity and 0% debt.[219, 220]

Table 11: Statistics Reported for 25 Size-Ranked Portfolios in Exhibits C-1 through C-8 (and not otherwise reported in the A Exhibits)

- Average debt to market value of equity.

- Arithmetic average historical unlevered risk premium over long-term Treasuries (average return on equity in excess of long-term Treasury bonds) since 1963. ($RP_{m+s,\ unlevered}$)

- "Smoothed" average historical unlevered risk premium: the fitted premium from a regression with the average historical unlevered risk premium as dependent variable and the logarithm of the average sorting criteria as independent variable ($RP_{m+s,\ unlevered}$)

  (The coefficients and constants from this regression analysis are in the top right hand corner of the exhibits)

- Average unlevered beta calculated using the "sum beta" method applied to monthly returns for 1963 through the latest year (see the *2013 SBBI Valuation Yearbook* pp. 79-80 for a description of the "sum beta" method).

- Operating Margin: The mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.

- Coefficient of Variation of Operating Margin: The standard deviation of operating margin over the prior five years divided by the average operating margin for the same years.

- Coefficient of Variation of Return on Book Value of Equity: The standard deviation of return on book equity for the prior five years divided by the mean return on book equity for the same years. Return on book equity is defined as net income before extraordinary items minus preferred dividends divided by book value of common equity.

The purpose of the "C" exhibits is to give users of the *Report* the information they need to compare their subject company to the average company in the guideline portfolio in which their company falls. For example, if the operating margin of the subject company is significantly *less* than the average operating margin of the companies that make up the guideline portfolio, then (all things held the same) this may be an indication that the subject company is *riskier* than the average company in the guideline portfolio (or vice versa). This analysis may indicate the *direction* of an adjustment (increase or decrease), but not the *magnitude* of adjustment needed.

Gauging the "magnitude" of the potential adjustment needed is easier said than done, simply because there is the potential for so much overlap between size risk and accounting-based fundamental risk factors (e.g., as size decreases, variability of earnings tends to increase, and vice versa (see Graphs 18(a) and Graph 18(b) on page 84). For now, the best one might hope for is establishing a range in which the adjustment likely falls.

The way to establish this range is straightforward: if the accounting-based "fundamental risk measure" of the subject company is significantly different than that of the guideline portfolio in which the subject company falls, then identify the guideline portfolio in the equivalent "D" exhibit which has the most similar fundamental risk measure. The *difference* in the smoothed risk premia for the guideline portfolio in which the subject company falls, and the smoothed risk premia for the guideline portfolio that has the most similar fundamental risk measure is arguably a likely range in which the adjustment falls.

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[219] The D exhibits also include "unlevered" risk premia, but these are unlevered versions of the corresponding "levered" risk premia found in the D exhibits. The unlevered premia in the C exhibits are unlevered versions of the corresponding "levered" risk premia found in the A exhibits.

[220] The unlevered risk premia over the risk-free rate found in the D exhibits ($RP_{m+u,unlevered}$) are used in example 6 (see page 96) to estimate cost of equity capital using *Risk Study* inputs.

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

Identifying the "equivalent C exhibit" to the "A" or "B" exhibit in which the subject company falls is easy. In the *Risk Premium Report*, the returns of the same 25 portfolios (sorted by various size measures) are used to estimate the "risk premia over the risk-free rate" found in the "A" exhibits and the "size premia" found in the "B" exhibits. The "comparative risk characteristics" reported in the "C" exhibits are the average risk characteristics of the companies in these *same* 25 size-ranked portfolios.[221] So, the "equivalent C exhibit" to use is summarized in Table 12:

Table 12: Identifying the Equivalent "C" Exhibit

	Buildup Model: Use "A" Exhibits		CAPM: Use "B" Exhibits		Equivalent "C" Exhibit
Market Value of Equity	A-1	or	B-1	→	C-1
Book Value of Equity	A-2	or	B-2	→	C-2
5-Year Average Net Income	A-3	or	B-3	→	C-3
MVIC	A-4	or	B-4	→	C-4
Total Assets	A-5	or	B-5	→	C-5
5-Year Average EBITDA	A-6	or	B-6	→	C-6
Sales	A-7	or	B-7	→	C-7
Number of Employees	A-8	or	B-8	→	C-8

While this may seem a little confusing, it is really no more complex than the example from earlier where the house with a two-car garage was probably more valuable than the "peer group" made up of houses with only one-car garages. A couple examples will illustrate this.

Example: Using the "C" Exhibits and the Buildup Method

Using data from the *2013 Risk Premium Report* and using the Buildup method, assume that the size of the subject company based on its 5-year average net income is $20 million. This places it in Portfolio 23 of Exhibit A-3[222], and the corresponding "smoothed average risk premium" over the risk-free rate (RP_{m+s}) from Exhibit A-3 is 11.3 percent.

Next, looking at Exhibit C-3 (the "equivalent C exhibit" to Exhibit A-3), we find that the average operating margin of the companies used to calculate the risk premium in the Portfolio 23 of Exhibit A-3 is 8.5 percent. If the subject company's operating margin is, say, 6.0 percent, it may be *riskier* than the average similarly-sized company in the Exhibit A-3's Portfolio 23, all things held the same. So, the analysis thus far indicates that the smoothed average risk premium to use is 11.3 percent, but that there may be justification for an upward adjustment since the subject company is a little different from the guideline portfolio (i.e., which is the "peer group" we are comparing it to), as measured by operating margin.

How much of an adjustment to the risk premium is indicated? Looking now to Exhibit D-1, find the portfolio that has the closest average operating margin compared to the subject company's operating margin (6.0%). This ends up being Portfolio 21, with an average operating margin of 6.1 percent, and a smoothed average risk premium of 11.60 percent (11.6 percent rounded). Finally, identify the portfolio in Exhibit D-1 that has an operating margin closest to 8.5 percent. This ends up being Portfolio 17, with an average operating margin of 8.8 percent, and a smoothed average risk premium of 10.28 percent (10.3 percent rounded). Now, as previously discussed, gauging the magnitude of the adjustment is "easier said than done" (because there is the potential for overlap between size and accounting-based fundamental risk factors), but it would be reasonable to say that this analysis may indicate:

- An increase (upward adjustment) from the smoothed average "risk premium over the risk-free rate" of Exhibit A-3's Portfolio 23 (11.3%) may be appropriate.

- This adjustment likely falls into a range of 0% to 1.3 percent, which is the difference between the smoothed average "risk premium over the risk-free rate" of Exhibit D-1's Portfolio 21 (11.6%) and the smoothed average "risk premium over the risk-free rate" of Exhibit D-3's Portfolio 17 (10.3%).

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[221] Exhibits A-1 through A-8 and B-1 through B-8 use the same respective size-ranked portfolios, but calculate different statistics for each exhibit. For example, the 25 portfolios ranked by "book value of equity" are used in Exhibit A-2 and Exhibit B-2, but risk premia over the risk-free rate (RP_{m+s}) for use in a buildup method are calculated for Exhibit A-2, while risk premia over CAPM (RP_s, or "size premia") for use in CAPM and Buildup 2 are calculated for Exhibit B-2.

[222] When using the "Buildup 1" method, use the "A" exhibits. The "A" exhibits provide risk premia that can be added to a risk-free rate in a buildup method to estimate cost of equity capital (COE).

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

Example: Using the "H-C" Exhibits and High-Financial-Risk Companies

The *Risk Study* provides analysis that correlates historical equity returns (and historical risk premiums) directly with three measures of company-specific risk derived from accounting information (five-year operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity). These may also be called "fundamental" measures of company risk to distinguish them from stock market-based measures of equity risk (e.g. beta). The *Risk Study* demonstrates that as company size decreases, measures of risk calculated from financial statement data do, as a matter of fact, tend to increase.

In the *High-Financial-Risk Study*, one measure of accounting-data-based fundamental risk (five-year operating income margin) was examined for portfolios formed by ranking public companies by z-Score (manufacturing companies) and z"-Score (service companies).[224, 225]

The H-C exhibits can be used to compare a subject company's operating margin to the operating margins of portfolios made up of companies with similar z-Scores (or z"-Scores). For example, in the previous examples (specifically, Example 7 and Example 8), the subject company was a manufacturing company with a z-Score of 1.47, placing it in the "distressed" zone in Exhibits H-A and H-B. The average operating margin (2.57%) of the companies comprising the portfolio used to calculate the statistics for "manufacturing" companies in the distress zone in Exhibits H-A and H-B is published in Exhibit H-C (see Figure 67).

If the hypothetical subject company in these examples has a higher operating margin of, say 7 percent, it may be less risky than companies with similar z-Scores. This may suggest that a downward company-specific risk adjustment is justified.

Figure 67: Exhibit H-C

Companies Ranked by Market Value of Equity: Comparative Risk Characteristics
High-Financial-Risk Company Data for Year Ending December 31, 2012

Portfolio Rank	Arithmetic Average Risk Premium	Average Debt toMVIC	Average Debt to Market Value of Equity	Beta (SumBeta) Since '63	Average Operating Margin
Manufacturing (z-Score)					
1.8 to 2.99 *(gray zone)*	14.32%	46.26%	86.09%	1.57	2.14%
< 1.8 *(distress zone)*	16.52%	57.76%	136.73%	1.66	**2.57%**
Service Industry (z"-Score)					
1.1 to 2.59 *(gray zone)*	20.22%	41.45%	70.79%	1.59	3.53%
< 1.1 *(distress zone)*	27.69%	48.98%	96.01%	1.72	2.30%

[224] Because the denominators of the other two ratios (coefficient of variation in operating income margin, and coefficient of variation in return on book equity) are often negative for companies in the high-financial-risk portfolios (as a result of either negative earnings or negative book value of equity), developing comparable "high-financial-risk" premia for them frequently results in meaningless statistics.

[225] Operating margin is defined here as the mean operating income for the prior five years divided by the mean sales for the prior five years. Operating income is defined as sales minus cost of goods sold plus selling, general, and administrative expenses plus depreciation.

The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report

Using the C Exhibits to Refine COE Estimates: Closing Thoughts

In this section, examples are provided that demonstrate how *Report* users can use the comparative risk characteristics found in the *Report's* "C" exhibits to better judge whether an increase or decrease is indicated from the default guideline portfolio risk premium or size premium, and also to gauge a possible range in which this adjustment likely falls. It is important to note that the methods described here are only intended to give probable direction and likely range of possible adjustments, and do *not* yield "absolute" adjustments.

Almost any analysis ultimately boils down to comparing various alternatives to each other and trying to weigh the similarities and differences of those alternatives. *The Duff & Phelps Risk Premium Report* includes information about the characteristics of the companies that comprise each of the size portfolios in the *Report*. That is arguably a better alternative than *not* having this information available, where one would simply accept the risk premium or size premium of the guideline portfolio, as is.

Appendix C

NOTE: Appendix A ("Definitions of Compustat Data Items Used in the *Report*") and Appendix B ("Changes to the *Report* Over Time") are not shown here.

Overview of Duff & Phelps U.S. ERP and Matching Risk Free Rate

The equity risk premium (ERP), sometimes referred to as the "market" risk premium, is defined as the return investors expect as compensation for assuming the additional risk associated with an investment in a diversified portfolio of common stocks in excess of the return they would expect from an investment in risk-free securities. The ERP is a key input used to calculate the cost of equity capital within the context of the Capital Asset Pricing Model (CAPM), the buildup model, and other models. Duff & Phelps regularly reviews fluctuations in global economic and financial conditions that warrant periodic reassessments of ERP. In this appendix, an overview of the Duff & Phelps U.S. ERP and matching risk free rate is provided.

To learn more about the Duff & Phelps U.S. ERP recommendation and matching risk-free rate, including a detailed discussion of adjusting cost of equity capital inputs during times of flight to quality and/or high levels of central bank intervention, please visit www.DuffandPhelps.com/CostofCapital *and download a free copy of "Client Alert: Duff & Phelps Decreases U.S. Equity Risk Premium Recommendation to 5.0%, Effective February 28, 2013".*

The Duff & Phelps Equity Risk Premium (ERP) Estimation Methodology is a Two-Dimensional Process

There is no single universally accepted methodology for estimating the Equity Risk Premium (ERP); consequently there is wide diversity in practice among academics and financial advisors with regards to recommended ERP estimates. For this reason, Duff & Phelps employs a two-dimensional process that takes into account a broad range of economic information and multiple ERP estimation methodologies to arrive at our recommendation.

Long-term research indicates that the ERP is cyclical. We use the term *normal*, or *unconditional* ERP to mean the long-term average ERP without regard to current market conditions. This concept differs from the *conditional* ERP, which reflects current economic conditions.[227] The "unconditional" ERP range versus a "conditional" ERP is further distinguished as follows:

"What is the range?"

- **Unconditional ERP Range –** The objective is to establish a reasonable range for a normal or unconditional ERP that can be expected over an entire business cycle. Based on the analysis of academic and financial literature and various empirical studies, we have concluded that a reasonable long-term estimate of the normal or unconditional ERP for the U.S. is in the range of 3.5% to 6.0%.[228]

"Where are we in the range?"

- **Conditional ERP –** The objective is to determine where within the unconditional ERP range the conditional ERP should be, based on current economic conditions. Research has shown that ERP is cyclical during the business cycle. When the economy is near (or in) a recession, the conditional ERP is at the higher end of the normal, or unconditional ERP range; conversely, when the economy improves, the conditional ERP reverts back toward the middle of the range. At the peak of an economic expansion, the conditional ERP migrates closer to the lower end of the range.

The Duff & Phelps ERP Recommendation is Currently Estimated in Conjunction with a Normalized Risk-Free Rate

All ERP estimates are, by definition, computed relative to a risk-free rate. In other words, the ERP is defined as the return investors expect as compensation for assuming the additional risk associated with an investment in a diversified portfolio of common stocks in excess of the return they would expect from an investment in risk-free securities. A risk-free rate is the return available on a security that the market generally regards as free of the risk of default.[229]

In developing our current U.S. ERP, Duff & Phelps matched this ERP with a "normalized" 20-year yield on U.S. government bonds of 4.0% as a proxy for the risk-free rate (R_f). Because investors expect that the returns from an investment in equities will be *at least as much* as the returns that they would receive from an investment in a risk-free asset, most of the widely used methods for estimating the cost of equity capital (e.g., build-up method, capital asset pricing model, and Fama-French three-factor model) *begin* with the yield-to-maturity on U.S. government securities (as of the valuation date), and then *build* upon that.

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[227] The "conditional" ERP is the estimate published by Duff & Phelps as the "Duff & Phelps Recommended ERP".

[228] See Shannon P. Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples*, Fourth Edition, Chapter 9,"Equity Risk Premium", pages 115–158 for a detailed discussion of the ERP.

[229] See Shannon P. Pratt and Roger J. Grabowski, *Cost of Capital: Applications and Examples,* Fourth Edition, Chapter 7,"Build-up Method", pages 88–94 for a detailed discussion of the risk-free rate.

Appendix C

Overview of Duff & Phelps U.S. ERP and Matching Risk Free Rate

Table 13 displays the spot 20-year U.S. Treasury yield-to-maturity versus a normalized 20-year Treasury yield on a monthly basis, beginning with December 2007 (before the onset of the 2008 Financial Crisis). Periods where Duff & Phelps suggests "normalization" may be appropriate are shaded in blue.[230]

Table 13: 20-year U.S. Treasury Yield (spot rate) versus 20-year Treasury Yield (normalized rate)

Date	20-year Treasury Yield (%) (spot rate)	20-year Treasury Yield (%) (normalized rate)	Date	20-year Treasury Yield (%) (spot rate)	20-year Treasury Yield (%) (normalized rate)
—	—	—	Jun-10	3.74	4.00
Dec-07	4.50	4.50	Jul-10	3.74	4.00
Jan-08	4.35	4.35	Aug-10	3.23	4.00
Feb-08	4.37	4.37	Sep-10	3.38	4.00
Mar-08	4.30	4.30	Oct-10	3.64	4.00
Apr-08	4.49	4.49	Nov-10	3.80	4.00
May-08	4.74	4.74	Dec-10	4.13	4.13
Jun-08	4.59	4.59	Jan-11	4.33	4.33
Jul-08	4.63	4.63	Feb-11	4.25	4.25
Aug-08	4.47	4.47	Mar-11	4.29	4.29
Sep-08	4.43	4.43	Apr-11	4.15	4.15
Oct-08	4.74	4.74	May-11	3.91	4.00
Nov-08	3.71	4.50	Jun-11	4.09	4.09
Dec-08	3.05	4.50	Jul-11	3.77	4.00
Jan-09	3.86	4.50	Aug-11	3.19	4.00
Feb-09	3.98	4.50	Sep-11	2.66	4.00
Mar-09	3.61	4.50	Oct-11	2.89	4.00
Apr-09	4.10	4.50	Nov-11	2.77	4.00
May-09	4.34	4.50	Dec-11	2.57	4.00
Jun-09	4.30	4.30	Jan-12	2.59	4.00
Jul-09	4.29	4.29	Feb-12	2.73	4.00
Aug-09	4.14	4.14	Mar-12	3.00	4.00
Sep-09	4.02	4.02	Apr-12	2.73	4.00
Oct-09	4.19	4.19	May-12	2.27	4.00
Nov-09	4.07	4.07	Jun-12	2.38	4.00
Dec-09	4.58	4.58	Jul-12	2.21	4.00
Jan-10	4.38	4.38	Aug-12	2.29	4.00
Feb-10	4.40	4.40	Sep-12	2.42	4.00
Mar-10	4.55	4.55	Oct-12	2.46	4.00
Apr-10	4.36	4.36	Nov-12	2.37	4.00
May-10	4.05	4.05	Dec-12	2.54	4.00

[230] Source of data in Table 13: The Board of Governors of the Federal Reserve System at www.federalreserve.gov/datadownload/. Values in Table 13 may differ slightly from those published in the *2012 Risk Premium Report*: Daily yields (as reported at month-end) are presented in the *2013 Report*, rather than the monthly series.

Appendix C
Overview of Duff & Phelps U.S. ERP and Matching Risk Free Rate

Duff & Phelps reviews its ERP and risk-free rate guidance on a regular basis. To ensure that you are always using the most up-to-date U.S. ERP guidance, visit www.DuffandPhelps.com/CostofCapital.

TO BE CLEAR: The Duff & Phelps U.S. ERP recommendation for use with year-end 2012 (December 31, 2012) valuations is 5.5%, matched with a "normalized" 20-year yield on U.S. government bonds of 4.0% as a risk-free rate (R_f). This implies a 9.5% (4.0% + 5.5%) "base" U.S. cost of equity capital estimate at year-end 2012.

Based on information available after January 1, 2013, as of February 28, 2013 (and thereafter, until further notice), Duff & Phelps lowered its ERP recommendation to 5.0%, coupled with a normalized 4.0% risk-free rate (implying a 9.0% base U.S. cost of equity capital).

TO BE CLEAR: the ERP that is recommended for use with valuations as of February 28, 2013 (and thereafter, until further notice) is 5.0%, coupled with a normalized risk-free rate of 4.0%.

Duff & Phelps' ERP recommendations and accompanying risk-free rates for all periods from 2008 through present are presented in Table 14. The U.S. ERP estimate was developed relative to a 20-year Treasury yield (either "spot" or "normalized"), as shown.

Table 14: Duff & Phelps Recommended ERP and Corresponding Risk-Free Rates
January 2008–Present

	Duff & Phelps Recommended ERP	Risk-Free Rate
Current ERP Guidance ✓ February 28, 2013 – UNTIL FURTHER NOTICE	5.0%	4.0% Normalized 20-year Treasury yield *
Year-end 2012 Guidance ⊙ December 31, 2012	5.5%	4.0% Normalized 20-year Treasury yield *
Change in ERP Guidance January 15, 2012 – February 27, 2013	5.5%	4.0% Normalized 20-year Treasury yield *
Change in ERP Guidance September 30, 2011 – January 14, 2012	6.0%	4.0% Normalized 20-year Treasury yield *
July 2011 – September 29, 2011	5.5%	4.0% Normalized 20-year Treasury yield *
June 1, 2011 – June 30, 2011	5.5%	Spot 20-year Treasury Yield
May 1, 2011 – May 31, 2011	5.5%	4.0% Normalized 20-year Treasury yield *
December 1, 2010 – April 30, 2011	5.5%	Spot 20-year Treasury Yield
June 1, 2010 – November 30, 2010	5.5%	4.0% Normalized 20-year Treasury yield *
Change in ERP Guidance December 1, 2009 – May 31, 2010	5.5%	Spot 20-year Treasury Yield
June 1, 2009 – November 30, 2009	6.0%	Spot 20-year Treasury Yield
November 1, 2008 – May 31, 2009	6.0%	4.5% Normalized 20-year Treasury yield *
Change in ERP Guidance October 27, 2008 – October 31, 2008	6.0%	Spot 20-year Treasury Yield
January 1, 2008 – October 26, 2008	5.0%	Spot 20-year Treasury Yield

* Normalized in this context means that in months where the risk-free rate is deemed to be abnormally low, a proxy for a longer-term sustainable risk-free rate is used.

Frequently Asked Questions (FAQ)

General Questions

What is the difference between the *Duff & Phelps Risk Premium Report* and the *Duff and Phelps Risk Premium Calculator*?

The *Duff & Phelps Risk Premium Report* (published since 1996) is designed to help finance professionals assess risk and more accurately estimate the cost of equity capital for purposes of business valuation, capital budgeting, feasibility studies and corporate finance decisions. The *Report* analyzes the relationship between equity returns and size (eight alternative size measures are analyzed, including the traditional market capitalization), and the relationship between equity returns and three accounting-based measures of fundamental risk (one measure of profitability and two measures of earnings variability are analyzed).

The *Duff & Phelps Risk Premium Calculator* is an online application developed in 2011 that uses the same trusted data and methodology that is published in the *Duff & Phelps Risk Premium Report*. The *Duff & Phelps Risk Premium Calculator* takes the *Duff & Phelps Risk Premium Report* to the next level by quickly delivering four cost of equity capital estimates using multiple models (including the capital asset pricing model (CAPM) and Buildup models), and an instantly-delivers a fully customizable Executive Summary in Microsoft Word format that includes sourcing, key inputs, and a concluded range of cost of equity capital estimates. In addition, a detailed record of all inputs, outputs, and calculations is exported to a "support and detail" Microsoft Excel workbook.[231]

The Duff & Phelps Risk Premium Report includes a *Size Study*, a *Risk Study*, and a *High-Financial-Risk Study*. What is the difference?

The *Size Study* analyzes the relationship between eight alternative measures of size and return (the eight size measures are market capitalization, book value of equity, 5-year average net income, market value of invested capital (MVIC), total assets, 5-year average EBITDA, sales, and number of employees. and return). "Risk premia over the risk-free rate" (located in the A Exhibits) and "size premia" (located in the B Exhibits) are then calculated for 25 size-ranked portfolios. These premia can then be used to develop cost of equity capital (COE) estimates using the buildup method and the CAPM method (see the *Size Study* for detailed examples of each).

The *Risk Study* analyzes the relationship between accounting-based fundamental risk measures and return (the three fundamental risk measures are operating margin, coefficient of variation in operating margin, and coefficient of variation in return on equity). "Risk premia over the risk-free rate" (located in the D Exhibits) are then calculated for 25 risk-ranked portfolios. These premia can then be used to develop cost of equity capital (COE) estimates using the buildup method (see the *Risk Study* for detailed examples).

The *High-Financial-Risk Study* analyzes the companies identified as high-financial-risk, and therefore excluded from the *Size* and *Risk* studies. "Risk premia over the risk-free rate" for high-financial-risk companies (located in the H-A Exhibits) and "size premia" for high-financial-risk companies (located in the H-B Exhibits) are then calculated for two portfolios ranked by the Altman z-Score. These premia can then be used to develop cost of equity capital (COE) estimates using the Buildup and CAPM methods (see the *High-Financial-Risk Study* for detailed examples).

Questions relating to the proper use of the *Duff & Phelps Risk Premium Report*

Which exhibits do I use to estimate COE using the "buildup" method?

The primary source of "buildup" risk premia in the Report is the *Size Study*'s Exhibits A-1 through A-8, which provide "risk premia over the risk-free rate". "Buildup" risk premia can also be found in the *Risk Study*'s Exhibits D-1 through D-3, and in the *High-Financial-Risk Study*'s Exhibit H-A[232]. Unlevered risk premia over the risk-free rate can be found in Exhibits C-1 through C-8.

A common characteristic of "risk premia over the risk-free rate" is that they are in terms of the combined effect of *market risk* and *size risk*, and are designed to be added to a risk-free rate to estimate COE. Another common characteristic of "risk premia over the risk-free rate" is that they *always* require the "ERP Adjustment" (see page 12 for more information on the ERP Adjustment).

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[231] The *Duff & Phelps Risk Premium Calculator* is available through Business Valuation Resources (BVR) and ValuSource.
[232] The risk premia over the risk-free rate found in Exhibit H-A are reproduced in Exhibit H-C.

Frequently Asked Questions (FAQ)

Which exhibits do I use to estimate COE using the "CAPM" method?

Beta is a key input of the CAPM, but the betas of smaller companies do not fully explain the returns of smaller companies. Hence, a common adjustment to the CAPM is an adjustment for "size". The primary source of "size premia" in the *Report* is the *Size Study*'s Exhibits B-1 through B-8. Size premia can also be found in the *High-Financial-Risk Study*'s Exhibit H-B.

A common characteristic of "size premia" is that they are "beta-adjusted". In other words market risk as measured by "beta" has been controlled for, or removed, leaving only the size effect's contribution to excess return. Another common characteristic of "size premia" is that they *never* require the "ERP Adjustment" (see page 12 for more information on the ERP Adjustment).

What is the difference between the "Guideline Portfolio Method" and the "Regression Method"?

The *Duff & Phelps Risk Premium Report* and accompanying online *Duff & Phelps Risk Premium Calculator* provide two ways for users to match their subject company's size (or risk) characteristics with the appropriate smoothed risk premium: the "guideline portfolio" method, and the "regression equation" method.

With the guideline portfolio method, one accepts the smoothed risk premium or size premium of the guideline portfolio. In other words, you identify which of the 25 portfolios the subject company falls into, and simply use the smoothed risk or size premium that is published for that portfolio. With the regression equation method, one uses the regression equations for the given exhibit to calculate an "exact" interpolated smoothed risk premia or size premia *between* the guideline portfolios. To learn more, see page 19.

Should I use the guideline portfolio method or the regression equation method?

Although the guideline portfolio is simpler and more direct, the more flexible regression equation method is the suggested method in most cases. The online *Duff & Phelps risk Premium Calculator* automatically calculates both methods.

Should I use "smoothed" or "average" risk premia and size premia?

Smoothing the premia essentially averages out the somewhat scattered nature of the raw average premia. The "smoothed" average risk premium is generally the most appropriate indicator for most of the portfolio groups. To learn more see "Using 'Smoothed' Premia versus Using 'Average' Premia" on page 17.

Can my subject company be too small to use the regression method?

The *Duff & Phelps Risk Premium Report* and accompanying online *Duff & Phelps Risk Premium Calculator* can be used for smaller companies. Sometimes the required rate of return for a company that is significantly smaller than the average size of even the smallest of the *Report*'s 25 portfolios is being estimated. In such cases, it may be appropriate to extrapolate the risk premium to smaller sizes using the regression equation method.

As a general rule, extrapolating a statistical relationship far beyond the range of the data used in the statistical analysis is not recommended. However, extrapolations for companies with size characteristics that are within the range of the smallest companies comprising the 25th portfolio are within reason. We do not recommend extrapolating in cases where all size measures of the subject company are less than the smallest company comprising the 25th portfolio, and one should never use those size measures for which the subject company's size is equal to zero or negative. The *Duff & Phelps Risk Premium Report* includes a description of the size characteristics of the 25th portfolio in Table 2 on page 22.

Frequently Asked Questions (FAQ)

Do I have to have all eight of the size measures (or all three of the risk measures) for my subject company in order to use the *Report*?

No. It would not be unusual for fewer than the maximum number of eight size measures or fewer than the maximum number of three risk measures to be used when estimating COE using the *Report*. When using the *Size Study*, the minimum number of size measures required is one. However, we do suggest using as many size measures as possible for best results. When using the *Risk Study*, a minimum of the most recent 3 years of information is required to get results for any one of the three measures of fundamental risk.

Should I use the mean or median of my resulting COE estimates?

The median estimate is generally preferred to the mean, although both should be included in a valuation report. The mean (i.e., average) estimate has the potential of being more heavily influenced by very large or very small outliers than the median (i.e., typical) estimate is.

Can the Duff & Phelps Report "C" Exhibits be used to further refine my COE estimates?

Yes. A differentiating capability of the *Duff & Phelps Risk Premium Report* is that it includes information about the characteristics of the companies that make up the portfolios that are used to calculate the risk premia and size premia published in the *Report*. This is an important capability because it enables *Report* users to potentially further refine their cost of equity estimate (COE) by gauging how "alike or different" the subject company is compared to the companies that make up the *Report*'s guideline portfolios.

The *Duff & Phelps Risk Premium Report*'s "C" Exhibits can be used to gauge whether an increase or decrease adjustment to a risk premium or size premium (and thus, COE) might be appropriate, based upon the "company-specific" differences of the subject company's fundamental risk and the average fundamental risk of companies that make up the portfolios from which the risk premia are derived. To learn more, see *"The "C" Exhibits – A Powerful Feature of the Duff & Phelps Risk Premium Report"* on page 112.

What is the Duff & Phelps Recommended ERP?

The Duff & Phelps Recommended ERP is developed by taking into account *multiple* ERP estimation methodologies to identify a reasonable "unconditional" range in which the true ERP likely exists. A broad range of current economic information is then analyzed to gauge *where* in this range the "conditional" ERP is.[233]

The reason for using multiple models is simple – there is no single universally accepted methodology for estimating the equity risk premium, and relying on any single model can be problematic. The Duff & Phelps Recommended ERP (and corresponding risk-free rates) from 2008 to present can be found in Table 14 on page 130.

Why is the D&P ERP lower than the Ibbotson ERP(s)?

Research suggests that the true U.S. ERP is likely in the range 3.5% to 6.0%.[234] The median "historical" Ibbotson ERP as calculated over the1926–present time horizon for the last 10 years is 7.0%, with a high of 7.2% and a low of 6.5%. In regards to the selection of ERP, a 2010 decision in the Delaware Court of Chancery rejected the use of the Ibbotson ERP of 7.1% put forth by one expert (and instead chose a lower estimate of 6%), citing the "…wealth of recent academic and professional writings that supports a lower ERP estimate…" that were put forth in the hearing.[235]

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[233] By "conditional" ERP we mean "considering current economic conditions".

[234] To learn more about the equity risk premium (ERP), see *Cost of Capital: Applications and Examples* 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", page 115.

[235] *Global GT LP and Global GT LTD v. Golden Telecom, Inc.*, April 23, 2010. To learn more about this decision, download a free copy of "Client Alert: Delaware Chancery Court Fails to Adopt the Morningstar/Ibbotson Historical Equity Risk Premium (ERP)" at www.DuffandPhelps.com/CostofCapital

Frequently Asked Questions (FAQ)

Which ERP should be used? Duff & Phelps Recommended ERP or Ibbotson's "historical" or "supply side" ERP?

We suggest using the Duff & Phelps recommended ERP. Duff & Phelps employs a two-dimensional process that takes into account a broad range of economic information and *multiple* ERP estimation models to arrive at an ERP recommendation. As discussed in the previous question, there is no single universally accepted methodology for estimating the equity risk premium, and relying on any *single* model can be problematic. For example, at the end of 2007 the historical ERP (as calculated over the time period 1926–2007) was 7.1 percent. In 2008, the S&P 500 declined 37 percent, volatility increased significantly[236], and the financial crisis was reaching a zenith, but at the end of 2008 the "historical" ERP (as calculated over the time period 1926–2008) decreased to 6.5 percent, the opposite of what one might expect just as risks were rising (see Graph 19).[237]

Graph 19: "Historical" ERP as calculated over the time period 1926–2007 and 1926–2008
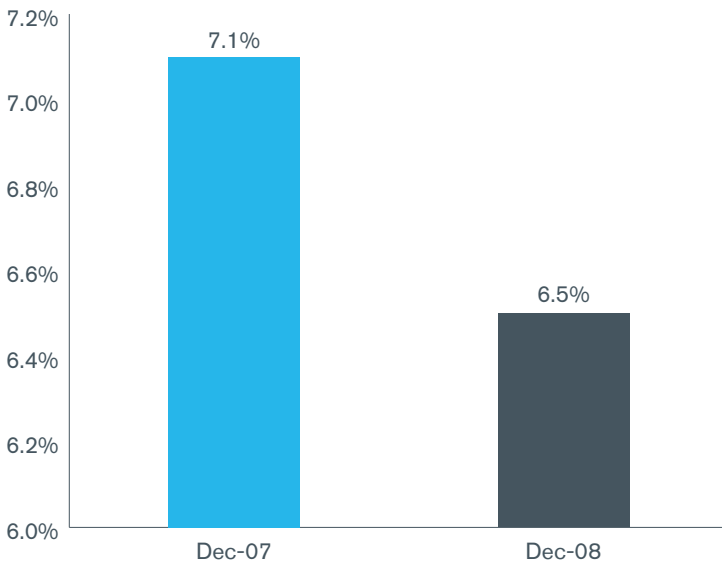
While "historical" models can be valid estimators of the expected (future) ERP to the degree that the past is expected to repeat itself, historical models can be sensitive to the time horizon chosen, may not adequately reflect possible changes in the relationships of equities and bonds over time, and may be influenced by non-market interventions.[238]

Ibbotson's "supply side" ERP is also primarily a historical model, but makes use of inputs typically supplied by companies: inflation, income return, and growth in real earnings. The model assumes that a fourth component, the price to earnings (PE) growth embedded in historical returns, is not sustainable and thus subtracts it out.[239] The Ibbotson supply side ERP is typically a lower estimate than Ibbotson's "historical" ERP. The majority of the analyses published in the *SBBI Yearbook*, including the size premiums on the SBBI "back page", are based on the higher historical ERP in the calculations. To learn more, see "Appendix C – Overview of Duff & Phelps U.S. ERP and Matchng Risk Free Rate" on page 130. To ensure you are always using the most up-to-date ERP and risk-free rate guidance from Duff & Phelps, visit www.DuffandPhelps.com/CostofCapital.

Should you adjust the risk-free rate, the equity risk premium or both in a changing economy?

In the aftermath of the 2008-2009 financial crisis, financial market conditions have changed dramatically in very short periods. During periods in which risk-free rates appear to be abnormally low due to flight to quality issues or other factors, one might consider either normalizing the risk-free rate or adjusting the equity risk premium (ERP). Duff & Phelps utilizes a combination of these options. Normalizing the risk-free rate is likely a more direct (and more easily implemented) analysis than adjusting the "conditional" equity risk premium (ERP) due to a *temporary* reduction in the yields on risk-free securities. Longer-term trends may be more appropriately reflected in the ERP. Duff & Phelps' ERP recommendations and accompanying risk-free rates for all periods from 2008 through present are presented in Table 14 in Appendix C.

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[236] As measured by the Chicago Board Options Exchange (CBOE) Volatility Index® (VIX®), which is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option price. The VIX Index rose from 22.5 on December 31, 2007 to 40.0 on December 31, 2008. The Vix reached a 2008 high of 80.9 on November 20.

[237] Calculated by Duff & Phelps. The "historical" long-horizon expected equity risk premium in this example is calculated as the average annual difference in SBBI Large Company Stocks (essentially the S&P 500 Index) and the income return of a 20-year U.S. Treasury bond. Source: Morningstar EnCorr.

[238] As paraphrased from Roger J. Grabowski, "Concerning the Equity Risk Premium and Structural Changes to Capital Markets", *Financial Valuation and Litigation Expert*, Issue 26, August/September 2010.

[239] *2011 SBBI Valuation Yearbook*, page 202 (Morningstar, Chicago, 2011)

Frequently Asked Questions (FAQ)

When is the ERP adjustment needed?

The ERP Adjustment accounts for the difference between the forward-looking ERP as of the valuation date that the *Report* user has selected to use in his or her COE calculations, and the historical (1963–present) ERP that was used as a *convention* in the calculations performed to create the *Report*. The ERP adjustment is only applicable in specific cases.

Although the online *Duff & Phelps Risk Premium Calculator* automatically calculates the ERP adjustment, properly applies it, and fully documents it, it is still important to understand the reasoning behind the adjustment. There are two basic ideas to remember in regards to when the ERP adjustment is necessary:

- The ERP adjustment is *always* necessary when using one of the *Report*'s "risk premia over the risk-free rate" (RP_{m+s}), because the historical ERP over the 1963–present time horizon for the SBBI Large Company Stocks index (essentially the S&P 500) is *embedded* in these premia. The *Report*'s risk premia over the risk-free rate (RP_{m+s}) come from Exhibits A-1 through A-8 (or from Appendix H-A, the "high-financial-risk" equivalent of the A exhibits), Exhibits C-1 through C-8, or Exhibits D-1 though D-3.

- The ERP adjustment is *never* necessary when using one of the *Report*'s "size premia", because the historical ERP over the 1963–present time horizon does *not* become embedded in size premia because size premia are beta adjusted (i.e., market risk adjusted). The *Report*'s size premia come from Exhibits B-1 through B-8 (or from Appendix H-B, the "high-financial-risk" equivalent of the B exhibits).

For a detailed discussion and examples of the ERP Adjustment, see *"Proper Application of the Equity Risk Premium (ERP) Adjustment"* on page 12.

Questions relating to the size effect

Is the size premium still a valid input?

While the size effect waxes and wanes, and may even be negative over significant portions of time, small company stocks' outperformance over large company stocks appears to be a persistent trend over the longer term. To learn more, see the discussion on the size effect on page 25.

Is the size premium really just a proxy for some other characteristic of smaller companies?

The idea that the size effect may be a proxy for "liquidity" or other risk factors included in the pricing of publicly traded stocks is not new. In a 1981 article often cited as the first comprehensive study of the size effect, Professor Rolf W. Banz[240] suggested as much, stating "It is not known whether size…is just a proxy for one or more true unknown factors correlated with size." More recent research by Abbott and Pratt; Ibbotson, Chen, and Hu; and others suggests that "liquidity" (a measure of the ease of transacting securities) may be what is actually being measured with the size effect. To learn more, see "Is the Size Effect a Proxy for Liquidity?" on page 39.

Questions relating to the online Duff & Phelps Risk Premium Calculator

What is the Duff & Phelps Risk Premium Calculator?

The *Duff & Phelps Risk Premium Calculator* is an online application developed in 2011 that uses the same trusted data and methodology that is published in the *Duff & Phelps Risk Premium Report*. The *Duff & Phelps Risk Premium Calculator* takes the *Duff & Phelps Risk Premium Report* to the next level by quickly delivering four cost of equity capital estimates using multiple models (including the capital asset pricing model (CAPM) and Buildup models), and an instantly-delivers a fully customizable "Executive Summary" in Microsoft Word format that includes sourcing, key inputs, and a concluded range of cost of equity capital estimates. In addition, a detailed record of all inputs, outputs, and calculations is exported to a "support and detail" Microsoft Excel workbook.[241]

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[240] Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics* (March 1981): 3–18.
[241] The *Duff & Phelps Risk Premium Calculator* is available through Business Valuation Resources (BVR) and ValuSource.

Frequently Asked Questions (FAQ)

How far back does the online Duff & Phelps Risk Premium Calculator data go?

With the online *Duff & Phelps Risk Premium Calculator* you can estimate cost of equity (COE) for any valuation date from January 1, 1996 to present (a total of 17+ years).

The *Risk Premium Calculator*'s underlying valuation database includes 17 years of risk premia and size premia for eight alternative measures of size (market capitalization, book value of equity, 5-year average net income, market value of invested capital (MVIC), total assets, 5-year average EBITDA, sales, and number of employees) and risk premia for three alternative measures of accounting-based fundamental risk factors (five-year average operating income margin, the coefficient of variation in operating income margin, and the coefficient of variation in return on book equity), and other important statistics, characteristics, and information.

Is the online Duff & Phelps Risk Premium Calculator easy to use?

Yes. The *Duff & Phelps Risk Premium Calculator* is very easy to use, and was designed specifically to help the growing number of users of the *Duff & Phelps Risk Premium Report* to efficiently and quickly get the most out of the methodology and data published in the *Report*, and to give them anywhere/anytime online access to the entire *Duff & Phelps Risk Premium Report*'s valuation database. After entering just a few basic inputs, the *Calculator* delivers an "Executive Summary" in Microsoft Word format that includes detailed results of up to four individual COE models, plus full "detail and support" of all inputs, calculations, and results in Microsoft Excel format.

The online Duff & Phelps Risk Premium Calculator automatically looks up a risk- free rate from the Federal Reserve Board of Governor's site.[242] Is this rate normalized?

No, this is the raw daily yield of a 20-year U.S. Treasury as of the valuation date entered. In most cases one would prefer to use the existing U.S. Treasury yield available in the market. However, during times of flight to quality or other factors' influence, a lower risk-free rate implies a lower cost of capital – the opposite of what one would expect in times of relative distress, and so a "normalization" adjustment may be indicated.

To learn more about the Duff & Phelps U.S. ERP and matching risk-free rate, including a detailed discussion of adjusting cost of equity capital inputs during times of flight to quality and/or high levels of central bank intervention, please visit please visit www.DuffandPhelps.com/CostofCapital and download a free copy of "*Client Alert: Duff & Phelps Decreases U.S. Equity Risk Premium Recommendation to 5.0%, Effective February 28, 2013*"

The Calculator asks for some inputs in millions. What would I enter for a smaller company with inputs less than a million?

If a subject company's size measure is less than a million the following table provides examples of how to input the correct amount:

	Total Assets	Net Sales	Net Sales
Subject Company	$5,500,000	$656,000	$96,000
Input (in millions)	**$5.500**	**$0.656**	**$0.096**

NOTE: all *Size Study*, *Risk Study*, and *High-Financial-Risk Study* inputs are in millions of dollars, with the exception of "Number of Employees", which is in standard units (i.e., if the subject company has 50 employees, enter "50", if the Subject Company has 200 employees, enter "200", etc.)

Does the online Duff & Phelps Risk Premium Calculator work for non-US-based companies?

The size data we have compiled in the *Duff & Phelps Risk Premium Report* is based on the U.S. market. In other words, it evaluates whether a company is small relative to large U.S. companies. Every market has a different benchmark for what constitutes a "large" or "small" size company. Our U.S. data may not be appropriate to measure size in other markets.

Does the online Duff & Phelps Risk Premium Calculator automatically make the ERP Adjustment?

Yes. The "ERP Adjustment" accounts for the difference between the forward-looking ERP as of the valuation date that the *Report* user has selected to use in his or her COE calculations, and the historical (1963–present) ERP that was used as a *convention* in the calculations performed to create the *Report*. The online *Duff & Phelps Risk Premium Calculator* calculates the appropriate ERP adjustment (based on the ERP the Report user has selected, and the valuation date). The *Calculator* then automatically applies the ERP Adjustment as necessary, and fully documents both the calculation and the application of the ERP Adjustment in the *Calculator*'s output documents (the Calculator's output documents include an "Executive Summary" in Microsoft Word format and a detailed record of all inputs, outputs, and calculations in a "Support and Detail" Microsoft Excel workbook). To learn more about the ERP Adjustment, see "*Proper Application of the Equity Risk Premium (ERP) Adjustment*" on page 12.

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[242] Source of U.S. 20-year constant maturity Treasury yields used in the online Duff & Phelps Risk Premium Calculator: www.federalreserve.gov/datadownload/

The Duff & Phelps
Risk Premium Calculator (web-based)

In 2011 we introduced the web-based *Duff & Phelps Risk Premium Calculator*. The *Calculator* automatically estimates levered and unlevered cost of equity capital (COE) for your subject company dependent on its size and risk characteristics (for any valuation date from January 1, 1996 to present), using both the capital asset pricing model (CAPM) and buildup models.

The *Calculator* is easy to use, saves time, and automatically provides full summary output in both Microsoft Word and Microsoft Excel format. In addition, the *Calculator* automatically looks up the long-term risk free rate for your valuation date[1], automatically makes the important (but often overlooked) "ERP Adjustment" to your subject company's COE estimates, and automatically adjusts an *SBBI* industry risk premium (IRP) so that it can be used in a Buildup model using *Risk Premium Report* size premia.[2]

Calculator Features

- Anytime, anywhere access at www.bvmarketdata.com/DP.RPC

- Complete historical database of risk premia and size premia data (1996 Report data to 2013 Report data)

- Automatic output
  - Executive Summary of COE estimates, including CAPM, Buildup, and unlevered COE
  - Microsoft Excel output of all underlying values and calculations

- Easy to use / Saves time

The *Calculator* employs the methodology and data published in the *Duff & Phelps Risk Premium Report*, which has provided financial and valuation professionals defensible cost of capital data and methodology since 1996[3]

Calculator Tour

Duff & Phelps designed the *Calculator* with two simple goals: the user experience had to be as easy and smooth as possible, and the *Calculator* had to maintain the same analytical horsepower, data, and methodology "under the hood" as is found in the *Risk Premium Report*.

There are three simple steps needed to calculate cost of equity capital (COE) using the *Calculator*.

Step 1	Step 2	Step 3

• Log in	• Enter Subject Company Inputs
	  • Size Characteristics
	  • Risk Characteristics
		• Receive Output
		  • Executive Summary
		  • Excel Summary

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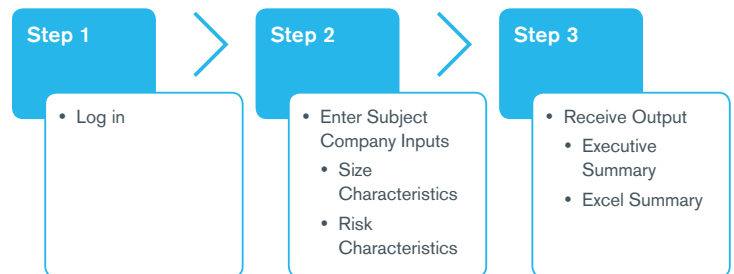
[1] 20-year constant maturity Treasury bond yield as of your valuation data. Source: The Board of Governors of the Federal Reserve System. These rates are nominal, and not "normalized". For more information about risk-free rate normalization, see the *2013 Duff & Phelps Risk Premium Report*. For historical 20-year nominal and normalized risk-free rates from 2008 to present, see Table 13 in the *2013 Report*.

[2] Duff & Phelps does not publish IRPs. A source of IRPs is Morningstar's *Ibbotson SBBI Valuation Yearbook*, (Chicago, Morningstar), Chapter 3, "The Buildup Method", Table 3-5.

[3] For detailed information about the *Size Study, Risk Study*, and *High-Financial-Risk Study* included in the *Risk Premium Report* (and now available in the *Risk Premium Calculator*), please see the *2013 Duff & Phelps Risk Premium Report*.

The Duff & Phelps
Risk Premium Calculator (web-based)

Step 1: Log in at www.bvmarketdata.com/DP.RPC

Image 1 – Logging in

Step 2a: Enter your subject company's name, and the valuation date.

Image 2 – Subject Company Name and Valuation Date

The Duff & Phelps
Risk Premium Calculator (web-based)

Step 2b: An optional set of questions and inputs is provided if the individual analyst has determined that the subject company is "high-financial-risk".[4]

Image 3 – Optional "High-Financial-Risk" Information

The five questions in this step mirror the five criteria by which high-financial-risk companies are identified in (and eliminated from) the universe of US companies to form the base set of companies used in the *Size Study* and *Risk Study*.

If you answer "Yes" to one or more of the five questions, it *may* suggest that the subject company's characteristics are more like the companies that make up the "high risk" portfolios rather than like the "healthy" companies that make up the standard 25 portfolios, but *not necessarily so*. For example, a company may have a debt to total capital ratio greater than 80%, but this does not automatically imply than the company is in distress.

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[4] The information and data in the *Duff & Phelps Risk Premium Calculator* is primarily designed to be used to develop cost of equity capital (COE) estimates for the large majority of companies that are fundamentally healthy, and for which a "going concern" assumption is appropriate. A set of "high-financial-risk" companies is set aside and analyzed separately in the *High-Financial-Risk Study*. The decision to apply a high-financial-risk premium is ultimately dependent on the analyst's professional judgment, based upon the analyst's detailed knowledge of the subject company. Please note that *High-Financial-Risk Study* output is available for calendar year 2010 valuation dates (and later) only.

The Duff & Phelps
Risk Premium Calculator (web-based)

Step 2c: The next step is entering your subject company's size characteristics and risk characteristics. Note that the appropriate long-term risk free rate in this case, (4.13%) for the valuation date is automatically looked up and entered in the "Risk Free Rate" field for your convenience.[5] If you want to use a different risk free rate, just type over the value that the *Calculator* automatically entered in this field.

Image 4 – Basic Inputs Screen (not filled out)

Also note that the *Calculator* provides information and tips which appear if you hover your mouse cursor over one of the information icons. These helpful tips provide quick assistance if you need the definition of an input, or the source of an input.

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[5] 20-year constant maturity Treasury bond yield as of your valuation data. Source: The Board of Governors of the Federal Reserve System. The risk free rate field can be overtyped (edited) by the analyst. These rates are nominal, and not "normalized". For more information about risk-free rate normalization, see the *2013 Duff & Phelps Risk Premium Report*. For historical 20-year nominal and normalized risk-free rates from 2008 to present, see Table 13 in the *2013 Report*.

The Duff & Phelps
Risk Premium Calculator (web-based)

Fill in your subject company's size characteristics and risk characteristics, as shown in Image 5.

Image 5 – Basic Inputs Screen (filled out)

Under "General Inputs", enter the equity risk premium (ERP) you want used in all cost of equity capital (COE) calculations. For example, many users of the *Risk Premium Report* use the *Duff & Phelps Recommended ERP*, which was 5.5 percent at the end of 2010.[6,7,8]

Also under "General Inputs", enter a beta if you would like COE estimated using the capital asset pricing (CAPM) model, and an industry risk premium (IRP) from the *SBBI Yearbook* if you would like COE estimated using a buildup model that utilizes an IRP to account for market risk.

Only one (of the eight total) *Size Study* inputs is required, but enter as many of the eight values as possible for best results.

If you wish to receive cost of equity capital estimates derived using the *Risk Study*, the three most recent years of information are required (for best results, enter the most recent five years of information).

Please note that the Calculator automatically makes the important (but often overlooked) "ERP Adjustment" to your subject company's COE estimates, and automatically adjusts an SBBI industry risk premium (IRP) so that it can be used in a Buildup model using *Risk Premium Report* size premia.

[6] For more information on the equity risk premium, see Cost of Capital: Applications and Examples 4th ed., by Shannon P. Pratt and Roger J. Grabowski (John Wiley & Sons, Inc., 2010), Chapter 9, "Equity Risk Premium", pages 115–158.

[7] See Roger J. Grabowski, "Developing the Cost of Equity Capital: Risk-Free Rate and ERP During Periods of 'Flight to Quality'". This paper will appear in the Business Valuation Review and can also be downloaded at Duff & Phelps' Cost of Capital site at www.DuffandPhelps/CostofCapital

[8] If no ERP is entered, the historical ERP as calculated over the time horizon 1963 to the (year of the valuation date -1) is used. For example, for a calendar year 2013 valuation date, if no ERP is entered by the analyst in "General Inputs" the ERP as calculated from 1963–2012 (4.5%) would be used in all calculations.

The Duff & Phelps
Risk Premium Calculator (web-based)

Prior to calculating COE estimates for your subject company, the *Calculator* displays a summary of all of your inputs as shown in Image 6. At this point you can review your inputs, and change them (if necessary).

By clicking the "Confirm" button, you are agreeing that all of your inputs are as you intend, and the *Calculator* then calculates cost of equity capital (COE) estimates for your subject company.

Image 6 – Confirm / Change Inputs

DUFF&PHELPS | Risk Premium Calculator

Please verify the information below and make any necessary changes.

Subject Company: ABC Widgets

Valuation Date: 12/31/2010

For your valuation date (December 31, 2010), the default report is the 2010 Duff & Phelps Risk Premium Report (data through December 31, 2009). You have chosen to use the 2011 Duff & Phelps Risk Premium Report (data through December 31, 2010).

You have indicated that the Subject Company, ABC Widgets, is NOT a financial services company (finance, insurance, or real estate).The Duff & Phelps Risk Premium Online Calculator is appropriately used to estimated cost of equity for non-financial firms, so it is OK TO PROCEED with cost of capital estimation for your Subject Company.

‹ change

High Financial Risk

You have indicated that your subject company is not "high financial risk".

‹ change

General Inputs
- You entered an Equity Risk Premium (ERP) of 5.5% to use in all cost of equity estimates.
- You entered a beta (β) of 1.20 to use in Capital Asset Pricing Model (CAPM) cost of equity estimates.
- You entered an IRP of 1.5%. An adjusted IRP of 1.2% will be used in the 'Buildup 2' cost of equity estimate. The adjusted IRP is calculated by: (IRP entered by the USER) x (the ERP entered by the USER) / (historical ERP (1926–2010)).

Size Study Inputs
- You entered a MARKET VALUE OF EQUITY of $120.00 million
- You entered a BOOK VALUE OF EQUITY of $100.00 million
- You entered a 5-YEAR AVERAGE NET INCOME of $10.00 million
- You entered a MARKET VALUE OF INVESTED CAPITAL (MVIC) of $180.00 million
- You entered TOTAL ASSETS of $300.00 million
- You entered 5-YEAR AVERAGE EBITDA of $30.00 million
- You entered a SALES of $700.00 million
- You entered 200 as the NUMBER OF EMPLOYEES

Risk Study Inputs
- You have entered enough information to calculate COE based upon the Risk Study.

‹ change

confirm ›

Calculator:
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Web site:
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Phone: (503) 291-7963

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What It's Worth

The Duff & Phelps
Risk Premium Calculator (web-based)

After the *Calculator* calculates estimates of the subject company's cost of equity capital (COE), an abbreviated online "results preview" is displayed, as shown in Image 7.

Image 7 – Cost of Equity Capital (COE) Estimates (online "results preview")

Your complete (as opposed to online "results preview") COE estimate report includes an "*Executive Summary*" in Microsoft Word format and a "*Support and Detail*" Microsoft Excel workbook, which can be instantly downloaded by clicking on the "**XLSX**" and "**DOCX**" links at the top of the online "results preview" page, as indicated in Image 7.

The Duff & Phelps
Risk Premium Calculator (web-based)

Your complete COE estimate report includes:

Executive Summary (in Microsoft Word format)

The *Executive Summary* is a high-level overview of data sourcing information, key inputs used in calculations, and cost of equity capital (COE) estimates for all models employed (with your subject company's information plugged into each model's equation)[9], plus a concluded range of COE estimates for your subject company (using both the *Size Study* and *Risk Study*).[10]

Because the *Executive Summary* is in Microsoft Word format, you can edit it and format it to suit your individual needs. For example, inserting your own disclaimer information or adding your company logo is easy.

***Support and Detail* summary of all inputs and calculations (in Microsoft Excel format)**

The *Support and Detail* workbook includes a summary of your subject company's size and fundamental risk characteristics (and all other inputs), and complete documentation of calculations and inputs for each of the models used to estimate cost of equity capital (COE) for your subject company.

The *Support and Detail* workbook also includes the data exhibits[11] for each of the guideline portfolios that match your subject company (by size and/or fundamental risk). This important information includes a complete listing of size premia and risk premia (both levered and unlevered), average arithmetic and geometric returns, sum betas, average debt to MVIC, average debt to market value, average operating margin, average coefficient of variation of operating margin, average coefficient of variation of ROE, z-Score, and more.

An additional (and very important) capability of the *Calculator* that is documented in the *Support and Detail* workbook is that the *Calculator* automatically maps your subject company's size measures from the *Size Study* to portfolios of companies sorted by the three fundamental risk measures analyzed in the *Risk Study*, and then analyzes whether an upward or downward "company-specific" risk adjustment is indicated for each of the three fundamental risk factors. Why is this important? If two or more of the indicators are saying the same thing (upward adjustment or downward adjustment), it is a very powerful argument in defending a company-specific risk adjustment.

An additional (and very important) capability of the *Calculator* that is documented in the *Support and Detail* workbook is that the *Calculator* automatically maps your subject company's size measures from the *Size Study* to portfolios of companies sorted by the three fundamental risk measures analyzed in the *Risk Study*, and then analyzes whether an upward or downward "company-specific" risk adjustment is indicated for each of the three fundamental risk factors. Why is this important? If two or more of the indicators are saying the same thing (upward adjustment or downward adjustment), it is a very powerful argument in defending a company-specific risk adjustment.

Because the *Support and Detail* workbook is in Microsoft Excel format, you can edit it and format it to suit your individual needs. The workbook also includes a table of content tab and section divider tabs, so that when printed it is an organized, polished document ready for insertion into your valuation engagement report as a detailed "support, sourcing, and documentation" section designed to accompany the *Executive Summary*.

For free samples of complete *Executive Summary* and *Support and Detail* outputs, or for more information about the *Calculator*, please visit:

www.BVResources.com/dp

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[9] Please note that the number of models employed is dependent on the completeness (or lack thereof) of subject company inputs entered by *Calculator* users.

[10] The *Duff & Phelps Risk Premium Calculator* is based upon the *Duff & Phelps Risk Premium Report*. The *Risk Premium Report* includes a *Size Study* (which analyzes the relationship between equity returns and company size using up to eight measures of company size), a *Risk Study* (which analyzes the relationship between equity returns and accounting-based fundamental risk measures), and a *High-Financial-Risk Study* (which analyzes the relationship between equity returns and high-financial-risk, as measured by the Altman z-Score).

[11] Exhibits A-1 through A-8 (used in the Buildup method); B-1 through B-8 (used in the CAPM method); C-1 through C-8 (used to compare your subject company's risk characteristics to portfolios comprised of companies of the same size as the subject company); D-1 through D-8 (used to estimate COE based upon fundamental risk factors); H-A, H-B, and H-C ("high-financial-risk" premia and size premia), and z-Score calculations. Please note that the data and information included in the *Support and Detail* workbook is dependent on the completeness (or lack thereof) of subject company inputs entered by *Calculator* users.

The Duff & Phelps
Risk Premium Calculator (web-based)

Product Purchasing Information

You can purchase the *Duff and Phelps Risk Premium Calculator* through Business Valuation Resources (BVR) at:

www.bvresources.com/dp

503-291-7963 ext. 2.

All purchases of the *Duff & Phelps Risk Premium Calculator* include a copy of the *Duff & Phelps Risk Premium Report*.

Calculator Option 1

Includes 18 years of size premia and risk premia data (1996 Report data to 2013 Report data): 1-year subscription includes a copy of the *2013 Duff & Phelps Risk Premium Report* and unlimited access to *Duff & Phelps Risk Premium Calculator* data from 1996-2013. Estimate cost of equity capital for any valuation date from January 1, 1996 to present. $499

Calculator Option 2

Single Year Duff & Phelps Risk Premium Report: Includes 1-time use of *Risk Premium Calculator*. $275

Data Exhibits

NOTE: The *Risk Premium Report's* data exhibits provide the risk premia and size premia that can be used to estimate cost of equity capital. The *Report's* data exhibits are not shown here, and are available in the full version *Report*.

The data exhibits include:

- **Exhibits A-1 through A-8:** The A exhibits provide risk premia over the risk-free rate in terms of the combined effect of *market* risk and *size* risk for 25 portfolios ranked by eight alternative measures of size (RP_{m+s}).

- **Exhibits B-1 through B-8:** The B exhibits provide risk premia over CAPM (i.e., "size premia") in terms of *size* risk for 25 portfolios ranked by eight alternative measures of size (RP_s).

- **Exhibits C-1 through C-8:** The C exhibits provide a "link" between the 25 size-ranked portfolios in the Report's "Size Study" A and B exhibits and the three accounting-based fundamental risk characteristics used in the *Report's* "Risk Study" D exhibits. These exhibits can be used to compare a subject company's fundamental risk characteristics to the fundamental risk characteristics of portfolios made up of similarly-sized companies.

- **Exhibits D-1, D-2, and D-3:** The D exhibits provide risk premia over the risk-free rate in terms of the combined effect of *market* risk and *company-specific* risk for 25 portfolios ranked by three alternative measures of fundamental risk (RP_{m+u}).

- **Exhibits H-A, H-B, and H-C:** The H exhibits provide "high-financial-risk" premia for portfolios ranked by Altman z-Score. These premia may be used in both buildup and CAPM estimates of cost of equity capital if the individual analyst has determined that the subject company is considered "high-financial-risk". Exhibit H-A is the high-financial-risk equivalent of the A exhibits, Exhibit H-B is the high-financial-risk equivalent of the B exhibits, and Exhibit H-C is the high-financial-risk equivalent of the C exhibits.

For more information, visit:
www.duffandphelps.com

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About Duff & Phelps

As a leading global independent provider of financial advisory and investment banking services, Duff & Phelps delivers trusted advice to our clients principally in the areas of valuation, transactions, financial restructuring, dispute and taxation. Our world class capabilities and resources, combined with an agile and responsive delivery, distinguish our clients'experience in working with us. With offices in North America, Europe and Asia, Duff & Phelps is committed to fulfilling its mission to protect, recover and maximize value for its clients.

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